



Preparation Manual

Mathematics/Physical Science/Engineering 6–12 (274)

Overview and Exam Framework

Reference Materials

Sample Selected-Response Questions

Sample Selected-Response Answers and Rationales

Preparation Manual

Section 3: Overview and Exam Framework Mathematics/Physical Science/Engineering 6–12 (274)

Exam Overview

Exam Name	Mathematics/Physical Science/Engineering 6–12
Exam Code	274
Time	5 hours
Number of Questions	135 selected-response questions
Format	Computer-administered test (CAT)

The TExES Mathematics/Physical Science/Engineering 6–12 (274) exam is designed to assess whether an examinee has the requisite knowledge and skills that an entry-level educator in this field in Texas public schools must possess. The 135 selected-response questions are based on the Mathematics/Physical Science/Engineering 6–12 exam framework and cover grades 6–12. The exam may contain questions that do not count toward the score. Your final scaled score will be based only on scored questions.

The Standards

Mathematics Standard

I

Number Concepts: The mathematics teacher understands and uses numbers, number systems and their structure, operations and algorithms, quantitative reasoning and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

Mathematics Standard

II

Patterns and Algebra: The mathematics teacher understands and uses patterns, relations, functions, algebraic reasoning, analysis and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

Mathematics Standard

III

Geometry and Measurement: The mathematics teacher understands and uses geometry, spatial reasoning, measurement concepts and principles and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

Mathematics Standard

IV

Probability and Statistics: The mathematics teacher understands and uses probability and statistics, their applications and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

**Mathematics Standard
V**

Mathematical Processes: The mathematics teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics and to communicate mathematically.

**Mathematics Standard
VI**

Mathematical Perspectives: The mathematics teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics and the evolving nature of mathematics and mathematical knowledge.

**Mathematics Standard
VII**

Mathematical Learning and Instruction: The mathematics teacher understands how children learn and develop mathematical skills, procedures and concepts; knows typical errors students make; and uses this knowledge to plan, organize and implement instruction to meet curriculum goals and to teach all students to understand and use mathematics.

**Mathematics Standard
VIII**

Mathematical Assessment: The mathematics teacher understands assessment and uses a variety of formal and informal assessment techniques appropriate to the learner on an ongoing basis to monitor and guide instruction and to evaluate and report student progress.

**Physical Science
Standard I**

The science teacher manages classroom, field and laboratory activities to ensure the safety of all students and the ethical care and treatment of organisms and specimens.

**Physical Science
Standard II**

The science teacher understands the correct use of tools, materials, equipment and technologies.

**Physical Science
Standard III**

The science teacher understands the process of scientific inquiry and its role in science instruction.

**Physical Science
Standard IV**

The science teacher has theoretical and practical knowledge about teaching science and about how students learn science.

**Physical Science
Standard V**

The science teacher knows the varied and appropriate assessments and assessment practices to monitor science learning.

**Physical Science
Standard VI**

The science teacher understands the history and nature of science.

**Physical Science
Standard VII**

The science teacher understands how science affects the daily lives of students and how science interacts with and influences personal and societal decisions.

**Physical Science
Standard VIII**

The science teacher knows and understands the science content appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in physical science.

**Physical Science
Standard XI**

The science teacher knows unifying concepts and processes that are common to all sciences.

Engineering Standard I

The beginning engineering teacher possesses a working knowledge of engineering fundamentals.

**Engineering Standard
II**

The beginning engineering teacher understands the design process and integrates mathematics, science, technology, engineering and other knowledge to design solutions to engineering problems.

**Engineering Standard
III**

The beginning engineering teacher understands the legal and ethical requirements of the engineering profession.

**Engineering Standard
IV**

The beginning engineering teacher understands the concept of teaming and demonstrates knowledge of careers in engineering and technology and the role of professional and student engineering organizations in career development.

**Engineering Standard
V**

The beginning engineering teacher understands the societal contexts of engineering and technology.

**Engineering Standard
VI**

The beginning engineering teacher understands and applies knowledge of tools, equipment, technologies and procedures used in the design and production of systems.

**Engineering Standard
VII**

The beginning engineering teacher understands the importance of professional development and knows how students learn and develop engineering skills and concepts and uses this knowledge to plan and implement effective classroom instruction and laboratory experiences to meet curricular goals.

**Engineering Standard
VIII**

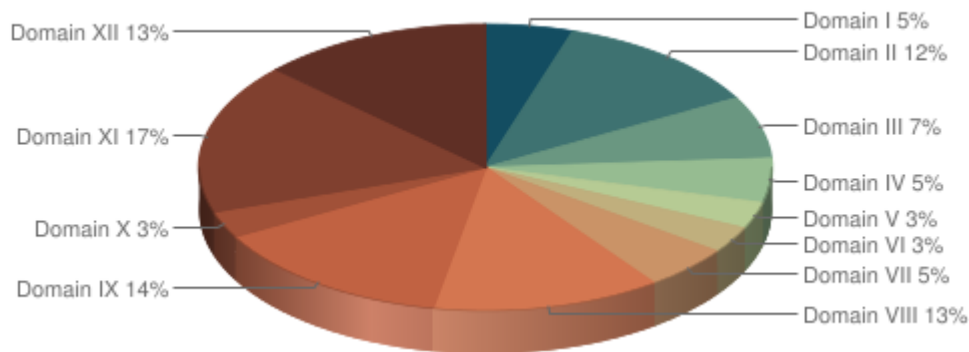
The beginning engineering teacher knows how to provide a safe and productive learning environment for implementing activities in engineering education.

**Engineering Standard
IX**

The beginning engineering teacher identifies, evaluates and utilizes new and emerging technologies.

Domains and Competencies

Domain	Domain Title	Approx. Percentage of Exam	Standards Assessed
I	Number Concepts	5%	Mathematics I
II	Patterns and Algebra	12%	Mathematics II
III	Geometry and Measurement	7%	Mathematics III
IV	Probability and Statistics	5%	Mathematics IV
V	Mathematical Processes and Perspectives	3%	Mathematics V—VI
VI	Mathematical Learning, Instruction and Assessment	3%	Mathematics VII—VIII
VII	Scientific Inquiry and Processes	5%	Physical Science I—III, VI—VII, XI
VIII	Physics	13%	Physical Science VIII
IX	Chemistry	14%	Physical Science VIII
X	Science Learning, Instruction and Assessment	3%	Physical Science IV—V
XI	The Engineering Method	17%	Engineering I—II, VI
XII	The Engineering Profession	13%	Engineering III—V, VII—IX



The content covered by this exam is organized into broad areas of content called **domains**. Each domain covers one or more of the educator standards for this field. Within each domain, the content is further defined by a set of **competencies**. Each competency is composed of two major parts:

- The **competency statement**, which broadly defines what an entry-level educator in this field in Texas public schools should know and be able to do.
- The **descriptive statements**, which describe in greater detail the knowledge and skills eligible for testing.

Domain I—Number Concepts

Competency 001—The teacher understands the real number system and its structure, operations, algorithms and representations.

The beginning teacher:

- A. Understands the concepts of place value, number base, and decimal representations of real numbers and rational numbers, including benchmark fractions.
- B. Understands the algebraic structure and properties of the real number system and its subsets (e.g., real numbers as a field, integers as an additive group, ordering of rational and real numbers).
- C. Describes and analyzes properties of subsets of real numbers (e.g., closure, identities).
- D. Selects and uses appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- E. Uses a variety of models (e.g., geometric, symbolic) to represent operations, algorithms and real numbers.
- F. Uses real numbers to model and solve a variety of problems.
- G. Uses deductive reasoning to simplify and justify algebraic processes.
- H. Demonstrates how some problems that have no solution in the integer or rational number systems have a solution in the real number system.

Competency 002—The teacher understands the complex number system and its structure, operations, algorithms and representations.

The beginning teacher:

- A. Demonstrates how some problems that have no solution in the real number system have a solution in the complex number system.
- B. Understands the properties of complex numbers (e.g., complex conjugate, magnitude/modulus, multiplicative inverse).
- C. Understands the algebraic structure of the complex number system and its subsets (e.g., complex numbers as a field, complex addition as vector addition).
- D. Selects and uses appropriate representations of complex numbers (e.g., vector, ordered pair, polar, exponential) for particular situations.
- E. Describes complex number operations (e.g., addition, multiplication, roots) using symbolic and geometric representations.

Competency 003—The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.

The beginning teacher:

- A. Applies ideas from number theory (e.g., prime numbers and factorization, the Euclidean algorithm, divisibility, congruence classes, modular arithmetic, the fundamental theorem of arithmetic) to solve problems.
- B. Applies number theory concepts and principles to justify and prove number relationships.

- C. Compares and contrasts properties of vectors and matrices with properties of number systems (e.g., existence of inverses, noncommutative operations).
- D. Uses properties of numbers (e.g., fractions, decimals, percents, ratios, proportions) to model and solve real-world problems.
- E. Applies counting techniques such as permutations and combinations to quantify situations and solve problems.
- F. Uses estimation techniques to solve problems and judge the reasonableness of solutions.

Domain II—Patterns and Algebra

Competency 004—The teacher uses patterns to model and solve problems and formulate conjectures.

The beginning teacher:

- A. Recognizes and extends patterns and relationships in data presented in tables, sequences or graphs.
- B. Uses methods of recursion and iteration to model and solve problems.
- C. Uses the principle of mathematical induction.
- D. Analyzes the properties of sequences and series (e.g., Fibonacci, arithmetic, geometric) and uses them to solve problems involving finite and infinite processes.
- E. Understands how sequences and series are applied to solve problems in the mathematics of finance (e.g., simple, compound and continuous interest rates; annuities).
- F. Determines the validity of logical arguments that include compound conditional statements by constructing truth tables.

Competency 005—The teacher understands attributes of functions, relations and their graphs.

The beginning teacher:

- A. Understands when a relation is a function.
- B. Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.
- C. Understands that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).
- D. Identifies and analyzes even and odd functions, one-to-one functions, inverse functions and their graphs.
- E. Applies basic transformations [e.g., $k f(x)$, $f(x) + k$, $f(x - k)$, $f(kx)$, $|f(x)|$] to a parent function, f , and describes the effects on the graph of $y = f(x)$.
- F. Performs operations (e.g., sum, difference, composition) on functions, finds inverse relations and describes results symbolically and graphically.
- G. Uses graphs of functions to formulate conjectures of identities [e.g., $y = x^2 - 1$ and $y = (x - 1)(x + 1)$, $y = \log x^3$ and $y = 3 \log x$, $y = \sin(x + \frac{\pi}{2})$ and $y = \cos x$].

Competency 006—The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

The beginning teacher:

- A. Understands the concept of slope as a rate of change, interprets the meaning of slope and intercept in a variety of situations and understands slope using similar triangles.
- B. Writes equations of lines given various characteristics (e.g., two points, a point and slope, slope and y -intercept).
- C. Applies techniques of linear and matrix algebra to represent and solve problems involving linear systems and uses arrays to efficiently manage large collections of data and add, subtract and multiply matrices to solve applied problems, including geometric transformations.
- D. Analyzes the zeros (real and complex) of quadratic functions.
- E. Makes connections between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ representations of a quadratic function and its graph.
- F. Solves problems involving quadratic functions using a variety of methods (e.g., factoring, completing the square, using the quadratic formula, using a graphing calculator).
- G. Models and solves problems involving linear and quadratic equations and inequalities using a variety of methods, including technology.

Competency 007—The teacher understands polynomial, rational, radical, absolute value and piecewise functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

The beginning teacher:

- A. Recognizes and translates among various representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value and piecewise functions.
- B. Describes restrictions on the domains and ranges of polynomial, rational, radical, absolute value and piecewise functions.
- C. Makes and uses connections among the significant points (e.g., zeros, local extrema, points where a function is not continuous or differentiable) of a function, the graph of the function and the function's symbolic representation.
- D. Analyzes functions in terms of vertical, horizontal and slant asymptotes.
- E. Analyzes and applies the relationship between inverse variation and rational functions.
- F. Solves equations and inequalities involving polynomial, rational, radical, absolute value and piecewise functions, using a variety of methods (e.g., tables, algebraic methods, graphs, use of a graphing calculator) and evaluates the reasonableness of solutions.
- G. Models situations using polynomial, rational, radical, absolute value and piecewise functions and solves problems using a variety of methods, including technology.
- H. Models situations using proportional and inverse variations, including describing physical laws such as Hook's law, Newton's second law of motion and Boyle's law.
- I. Uses precision and accuracy in real-life situations related to measurement and significant figures.
- J. Applies and analyzes published ratings, weighted averages and indices to make informed decisions.
- K. Uses proportionality to solve problems involving quantities that are not easily measured.

Competency 008—The teacher understands exponential and logarithmic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

The beginning teacher:

- A. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of exponential and logarithmic functions.
- B. Recognizes and uses connections among significant characteristics (e.g., intercepts, asymptotes) of a function involving exponential or logarithmic expressions, the graph of the function and the function's symbolic representation.
- C. Understands the relationship between exponential and logarithmic functions and uses the laws and properties of exponents and logarithms to simplify expressions and solve problems.
- D. Uses a variety of representations and techniques (e.g., numerical methods, tables, graphs, analytic techniques, graphing calculators) to solve equations, inequalities and systems involving exponential and logarithmic functions.
- E. Models and solves problems involving exponential growth and decay.
- F. Uses logarithmic scales (e.g., Richter, decibel) to describe phenomena and solve problems.
- G. Uses exponential and logarithmic functions to model and solve problems involving the mathematics of finance (e.g., compound interest).
- H. Uses the exponential function to model situations and solve problems in which the rate of change of a quantity is proportional to the current amount of the quantity [i.e., $f'(x) = k f(x)$].

Competency 009—The teacher understands trigonometric and circular functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

The beginning teacher:

- A. Analyzes the relationships among the unit circle in the coordinate plane, circular functions and trigonometric functions.
- B. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of trigonometric functions and their inverses.
- C. Recognizes and uses connections among significant properties (e.g., zeros, axes of symmetry, local extrema) and characteristics (e.g., amplitude, frequency, phase shift) of trigonometric functions, the graphs of functions and the functions' symbolic representations.
- D. Understands the relationships between trigonometric functions and their inverses and uses those relationships to solve problems.
- E. Uses trigonometric identities to simplify expressions and solve equations.
- F. Models and solves a variety of problems (e.g., analyzing periodic phenomena) using trigonometric functions.
- G. Uses graphing calculators to analyze and solve problems involving trigonometric functions.

Competency 010—The teacher understands and solves problems using differential and integral calculus.

The beginning teacher:

- A. Understands the concept of limit and the relationship between limits and continuity.
- B. Relates the concepts of proportionality, rates and average rate of change and applies those concepts to the slope of the secant line and the concept of instantaneous rate of change to the slope of the tangent line.
- C. Uses the first and second derivatives to analyze the graph of a function (e.g., local extrema, concavity, points of inflection).
- D. Understands and applies the fundamental theorem of calculus and the relationship between differentiation and integration.
- E. Models and solves a variety of problems (e.g., velocity, acceleration, optimization, related rates, work, center of mass) using differential and integral calculus.
- F. Analyzes how technology can be used to solve problems and illustrate concepts involving differential and integral calculus.

Domain III—Geometry and Measurement

Competency 011—The teacher understands measurement as a process.

The beginning teacher:

- A. Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.
- B. Applies formulas for perimeter, area, surface area and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.
- C. Recognizes the effects on length, area or volume when the linear dimensions of plane figures or solids are changed.
- D. Applies the Pythagorean theorem, proportional reasoning and right triangle trigonometry to solve measurement problems.
- E. Relates the concept of area under a curve to the limit of a Riemann sum.
- F. Uses integral calculus to compute various measurements associated with curves and regions (e.g., area, arc length) in the plane and measurements associated with curves, surfaces and regions in three-space.

Competency 012—The teacher understands geometries, in particular Euclidian geometry, as axiomatic systems.

The beginning teacher:

- A. Understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).
- B. Uses properties of points, lines, planes, angles, lengths and distances to solve problems.
- C. Applies the properties of parallel and perpendicular lines to solve problems.
- D. Uses properties of congruence and similarity to explore geometric relationships, justify conjectures and prove theorems.
- E. Describes and justifies geometric constructions made using compass and straightedge, reflection devices and other appropriate technologies.
- F. Demonstrates an understanding of the use of appropriate software to explore attributes of geometric figures and to make and evaluate conjectures about geometric relationships.
- G. Compares and contrasts the axioms of Euclidean geometry with those of non-Euclidean geometry (i.e., hyperbolic and elliptic geometry).

Competency 013—The teacher understands the results, uses and applications of Euclidian geometry.

The beginning teacher:

- A. Analyzes the properties of polygons and their components.
- B. Analyzes the properties of circles and the lines that intersect them.
- C. Uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).
- D. Computes the perimeter, area and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc length, area of sectors).
- E. Analyzes cross sections and nets of three-dimensional shapes.
- F. Uses top, front, side and corner views of three-dimensional shapes to create complete representations and solve problems.
- G. Applies properties of two- and three-dimensional shapes to solve problems across the curriculum and in everyday life, including in art, architecture and music.
- H. Uses similarity, geometric transformations, symmetry and perspective drawings to describe mathematical patterns and structure in architecture.

- I. Uses scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and nonproportional changes in surface area and volume as applied to fields.
- J. Uses the Pythagorean theorem and special right-triangle relationships to calculate distances.
- K. Uses trigonometric ratios to calculate distances and angle measures as applied to fields, including using models of periodic behavior in art and music.
- L. Solves geometric problems involving indirect measurement, including similar triangles, the Pythagorean theorem, law of sines, law of cosines and the use of dynamic geometry software.

Competency 014—The teacher understands coordinate, transformational and vector geometry and their connections.

The beginning teacher:

- A. Identifies transformations (i.e., reflections, translations, glide reflections, rotations, and dilations) and explores their properties.
- B. Uses the properties of transformations and their compositions to solve problems.
- C. Uses transformations to explore and describe reflectional, rotational and translational symmetry.
- D. Applies transformations in the coordinate plane.
- E. Applies concepts and properties of slope, midpoint, parallelism, perpendicularity and distance to explore properties of geometric figures and solve problems in the coordinate plane.
- F. Uses coordinate geometry to derive and explore the equations, properties and applications of conic sections (i.e., lines, circles, hyperbolas, ellipses, parabolas).
- G. Relates geometry and algebra by representing transformations as matrices and uses this relationship to solve problems.
- H. Explores the relationship between geometric and algebraic representations of vectors and uses this relationship to solve problems.

Domain IV—Probability and Statistics

Competency 015—The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.

The beginning teacher:

- A. Selects and uses an appropriate measurement scale (i.e., nominal, ordinal, interval, ratio) to answer research questions and analyze data.
- B. Organizes, displays and interprets data in a variety of formats (e.g., tables, frequency distributions, scatterplots, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).
- C. Applies concepts of center, spread, shape and skewness to describe a data distribution.

- D. Understands measures of central tendency (i.e., mean, median, and mode) and dispersion (i.e., range, interquartile range, variance, standard deviation).
- E. Applies linear transformations (i.e., translating, stretching, shrinking) to convert data and describes the effect of linear transformations on measures of central tendency and dispersion.
- F. Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers and measures of central tendency and dispersion.
- G. Supports arguments, makes predictions and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.

Competency 016—The teacher understands concepts and applications of probability.

The beginning teacher:

- A. Understands how to explore concepts of probability through sampling, experiments and simulations and generates and uses probability models to represent situations.
- B. Uses the concepts and principles of probability to describe the outcomes of simple and compound events.
- C. Determines probabilities by constructing sample spaces to model situations; uses a two-way frequency table as a sample space to identify whether two events are independent and to interpret the results; calculates expected value to analyze mathematical fairness, payoff and risk.
- D. Solves a variety of probability problems using combinations, permutations, and solves problems involving large quantities using combinatorics.
- E. Solves a variety of probability problems using ratios of areas of geometric regions.
- F. Calculates probabilities using the axioms of probability and related theorems and concepts (i.e., addition rule, multiplication rule, conditional probability, independence).
- G. Understands expected value, variance and standard deviation of probability distributions (e.g., binomial, geometric, uniform, normal).
- H. Applies concepts and properties of discrete and continuous random variables to model and solve a variety of problems involving probability and probability distributions (e.g., binomial, geometric, uniform, normal).

Competency 017—The teacher understands the relationships among probability theory, sampling and statistical inference, and how statistical inference is used in making and evaluating predictions.

The beginning teacher:

- A. Applies knowledge of designing, conducting, analyzing and interpreting statistical experiments to investigate real-world problems.
- B. Analyzes and interprets statistical information (e.g., the results of polls and surveys) and recognizes misleading as well as valid uses of statistics.
- C. Understands random samples and sample statistics (e.g., the relationship between sample size and confidence intervals, biased or unbiased estimators).
- D. Makes inferences about a population using binomial, normal and geometric distributions.

- E. Describes, calculates and analyzes bivariate data using various techniques (e.g., scatterplots, regression lines, outliers, residual analysis, correlation coefficients).
- F. Understands how to transform nonlinear data into linear form in order to apply linear regression techniques to develop exponential, logarithmic and power regression models.
- G. Uses the law of large numbers and the central limit theorem in the process of statistical inference.
- H. Estimates parameters (e.g., population mean and variance) using point estimators (e.g., sample mean and variance).
- I. Understands the principles of hypotheses testing.
- J. Determines the number of ways an event may occur using combinations, permutations and the fundamental counting principle.
- K. Compares theoretical to empirical probability.
- L. Uses experiments to determine the reasonableness of a theoretical model (i.e., binomial, geometric).
- M. Identifies limitations and lack of relevant information in studies reporting statistical information, especially when studies are reported in condensed form.
- N. Interprets and compares statistical results using appropriate technology given a margin of error.
- O. Identifies the variables to be used in a study.
- P. Analyzes possible sources of data variability, including those that can be controlled and those that cannot be controlled.
- Q. Reports results of statistical studies to a particular audience by selecting an appropriate presentation format, creating graphical data displays and interpreting results in terms of the question studied.

Domain V—Mathematical Processes and Perspectives

Competency 018—The teacher understands mathematical reasoning and problem solving.

The beginning teacher:

- A. Understands the nature of proof, including indirect proof, in mathematics.
- B. Applies correct mathematical reasoning to derive valid conclusions from a set of premises.
- C. Uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.
- D. Uses formal and informal reasoning to justify mathematical ideas.
- E. Understands the problem-solving process (i.e., recognizing that a mathematical problem can be solved in a variety of ways, selecting an appropriate strategy, evaluating the reasonableness of a solution).
- F. Evaluates how well a mathematical model represents a real-world situation.

Competency 019—The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.

The beginning teacher:

- A. Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of a circle as a quadratic function of the radius, probability as the ratio of two areas, area of a plane region as a definite integral).
- B. Understands how mathematics is used to model and solve problems in other disciplines (e.g., art, music, science, social science, business).
- C. Translates mathematical ideas between verbal and symbolic forms.
- D. Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, and concrete).
- E. Understands the use of visual media (e.g., graphs, tables, diagrams, animations) to communicate mathematical information.
- F. Uses appropriate mathematical terminology to express mathematical ideas.
- G. Explores and applies concepts of financial literacy as it relates to teaching students (e.g., describes the basic purpose of financial institutions, distinguishes the difference between gross income and net income, identifies various savings options, defines different types of taxes, identifies the advantages and disadvantages of different methods of payment).
- H. Applies mathematics to model and solve problems to manage financial resources effectively for lifetime financial security (e.g., distinguishes between fixed and variable expenses, calculates profit in a given situation, develops a system for keeping and using financial records, describes actions that might be taken to balance a budget when expenses exceed income, balances a simple budget).
- I. Analyzes various voting and selection processes to compare results in given situations; selects and applies an algorithm of interest to solve real-life problems (e.g., using recursion or iteration to calculate population growth or decline, fractals or compound interest; determining validity in recorded and transmitted data using checksums and hashing; evaluating sports rankings, weighted class rankings and search-engine rankings; solving problems involving scheduling or routing using vertex-edge graphs, critical paths, Euler paths or minimal spanning trees); and communicates to peers the application of the algorithm in precise mathematical and nontechnical language.
- J. Determines or analyzes an appropriate cyclical model for problem situations that can be modeled with periodic functions; determines or analyzes an appropriate piecewise model for problem situations; creates, represents and analyzes mathematical models for various types of income calculations to determine the best option for a given situation; creates, represents and analyzes mathematical models for expenditures, including those involving credit, to determine the best option for a given situation; creates, represents and analyzes mathematical models and appropriate representations, including formulas and amortization tables, for various types of loans and investments to determine the best option for a given situation.

Domain VI—Mathematical Learning, Instruction and Assessment

Competency 020—The teacher understands how children learn mathematics and plans, organizes and implements instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

The beginning teacher:

- A. Applies research-based theories of learning mathematics to plan appropriate instructional activities for all students.
- B. Understands how students differ in their approaches to learning mathematics.
- C. Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.
- D. Understands how learning may be enhanced through the use of manipulatives, technology and other tools (e.g., stopwatches, scales, rulers).
- E. Understands how to provide instruction along a continuum from concrete to abstract.
- F. Understands a variety of instructional strategies and tasks that promote students' abilities to do the mathematics described in the TEKS.
- G. Understands how to create a learning environment that provides all students, including English-language learners, with opportunities to develop and improve mathematical skills and procedures.
- H. Understands a variety of questioning strategies to encourage mathematical discourse and help students analyze and evaluate their mathematical thinking.
- I. Understands how to relate mathematics to students' lives and a variety of careers and professions.

Competency 021—The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.

The beginning teacher:

- A. Understands the purpose, characteristics and uses of various assessments in mathematics, including formative and summative assessments.
- B. Understands how to select and develop assessments that are consistent with what is taught and how it is taught.
- C. Understands how to develop a variety of assessments and scoring procedures consisting of worthwhile tasks that assess mathematical understanding, common misconceptions and error patterns.
- D. Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor and modify instruction to improve mathematical learning for all students, including English-language learners.

Domain VII—Scientific Inquiry and Processes

Competency 022—The teacher understands how to select and manage learning activities to ensure the safety of all students and the correct use and care of organisms, natural resources, materials, equipment and technologies.

The beginning teacher:

- A. Uses current sources of information about laboratory safety, including safety regulations and guidelines for the use of science facilities.
- B. Recognizes potential safety hazards in the laboratory and in the field and knows how to apply procedures, including basic first aid, for responding to accidents.
- C. Employs safe practices in planning, implementing and managing all instructional activities and designs and implements rules and procedures to maintain a safe learning environment.
- D. Understands procedures for selecting, maintaining and safely using chemicals, tools, technologies, materials, specimens and equipment, including procedures for the recycling, reuse and conservation of laboratory resources and for the safe handling and ethical treatment of organisms.
- E. Knows how to use appropriate equipment and technology (e.g., Internet, spreadsheet, calculator) for gathering, organizing, displaying and communicating data in a variety of ways (e.g., charts, tables, graphs, diagrams, maps, satellite images, written reports, oral presentations).
- F. Understands how to use a variety of tools, techniques and technology to gather, organize and analyze data and perform calculations and knows how to apply appropriate methods of statistical measures and analysis.
- G. Knows how to apply techniques to calibrate measuring devices and understands concepts of precision, accuracy and error with regard to reading and recording numerical data from scientific instruments (e.g., significant figures).
- H. Uses the International System of Units (i.e., metric system) and performs unit conversions within and across measurement systems.

Competency 023—The teacher understands the nature of science, the process of scientific inquiry and the unifying concepts that are common to all sciences.

The beginning teacher:

- A. Understands the nature of science, the relationship between science and technology, the predictive power of science and limitations to the scope of science (i.e., the types of questions that science can and cannot answer).
- B. Knows the characteristics of various types of scientific investigations (e.g., descriptive studies, controlled experiments, comparative data analysis) and how and why scientists use different types of scientific investigations.
- C. Understands principles and procedures for designing and conducting a variety of scientific investigations, with emphasis on inquiry-based investigations, and knows how to communicate and defend scientific results.
- D. Understands how logical reasoning, verifiable observational and experimental evidence and peer review are used in the process of generating and evaluating scientific knowledge.
- E. Understands how to identify potential sources of error in an investigation, evaluate the validity of scientific data and develop and analyze different explanations for a given scientific result.

- F. Knows the characteristics and general features of systems, how properties and patterns of systems can be described in terms of space, time, energy and matter, and how system components and different systems interact.
- G. Knows how to apply and analyze the systems model (e.g., interacting parts, boundaries, input, output, feedback, subsystems) across the science disciplines.
- H. Understands how shared themes and concepts (e.g., systems, order and organization; evidence, models and explanation; change, constancy and measurements; evolution and equilibrium; and form and function) provide a unifying framework in science.
- I. Understands the difference between a theory and a hypothesis, how models are used to represent the natural world and how to evaluate the strengths and limitations of a variety of scientific models (e.g., physical, conceptual, mathematical).

Competency 024—The teacher understands the history of science, how science impacts the daily lives of students and how science interacts with and influences personal and societal decisions.

The beginning teacher:

- A. Understands the historical development of science, key events in the history of science and the contributions that diverse cultures and individuals of both genders have made to scientific knowledge.
- B. Knows how to use examples from the history of science to demonstrate the changing nature of scientific theories and knowledge (i.e., that scientific theories and knowledge are always subject to revision in light of new evidence).
- C. Knows that science is a human endeavor influenced by societal, cultural and personal views of the world and that decisions about the use and direction of science are based on factors such as ethical standards, economics and personal and societal biases and needs.
- D. Understands the application of scientific ethics to the conducting, analyzing and publishing of scientific investigations.
- E. Applies scientific principles to analyze factors (e.g., diet, exercise, personal behavior) that influence personal and societal choices concerning fitness and health (e.g., physiological and psychological effects and risks associated with the use of substances and substance abuse).
- F. Applies scientific principles, the theory of probability and risk/benefit analysis to analyze the advantages of, disadvantages of or alternatives to a given decision or course of action.
- G. Understands the role science can play in helping resolve personal, societal and global issues (e.g., recycling, population growth, disease prevention, resource use, evaluating product claims).

Domain VIII—Physics

Competency 025—The teacher understands the description of motion in one and two dimensions.

The beginning teacher:

- A. Generates, analyzes and interprets graphs describing the motion of a particle.
- B. Applies vector concepts to displacement, velocity and acceleration in order to analyze and describe the motion of a particle.
- C. Solves problems involving uniform and accelerated motion using scalar (e.g., speed) and vector (e.g., velocity) quantities.
- D. Analyzes and solves problems involving projectile motion.
- E. Analyzes and solves problems involving uniform circular and rotary motion.
- F. Understands motion of fluids.
- G. Understands motion in terms of frames of reference and relativity concepts.

Competency 026—The teacher understands the laws of motion.

The beginning teacher:

- A. Identifies and analyzes the forces acting in a given situation and constructs a free-body diagram.
- B. Solves problems involving the vector nature of force (e.g., resolving forces into components, analyzing static or dynamic equilibrium of a particle).
- C. Identifies and applies Newton's laws to analyze and solve a variety of practical problems (e.g., properties of frictional forces, acceleration of a particle on an inclined plane, displacement of a mass on a spring, forces on a pendulum).

Competency 027—The teacher understands the concepts of gravitational and electromagnetic forces in nature.

The beginning teacher:

- A. Applies the law of universal gravitation to solve a variety of problems (e.g., determining the gravitational fields of the planets, analyzing properties of satellite orbits).
- B. Calculates electrostatic forces, fields and potentials.
- C. Understands the properties of magnetic materials and the molecular theory of magnetism.
- D. Identifies the source of the magnetic field and calculates the magnetic field for various simple current distributions.
- E. Analyzes the magnetic force on charged particles and current-carrying conductors.
- F. Understands induced electric and magnetic fields and analyzes the relationship between electricity and magnetism.
- G. Understands the electromagnetic spectrum and the production of electromagnetic waves.

Competency 028—The teacher understands applications of electricity and magnetism.

The beginning teacher:

- A. Analyzes common examples of electrostatics (e.g., a charged balloon attached to a wall, behavior of an electroscope, charging by induction).
- B. Understands electric current, resistance and resistivity, potential difference, capacitance and electromotive force in conductors and circuits.
- C. Analyzes series and parallel DC circuits in terms of current, resistance, voltage and power.
- D. Identifies basic components and characteristics of AC circuits.
- E. Understands the operation of an electromagnet.
- F. Understands the operation of electric meters, motors, generators and transformers.

Competency 029—The teacher understands the conservation of energy and momentum.

The beginning teacher:

- A. Understands the concept of work.
- B. Understands the relationships among work, energy and power.
- C. Solves problems using the conservation of mechanical energy in a physical system (e.g., determining potential energy for conservative forces, conversion of potential to kinetic energy, analyzing the motion of a pendulum).
- D. Applies the work-energy theorem to analyze and solve a variety of practical problems (e.g., finding the speed of an object given its potential energy, determining the work done by frictional forces on a decelerating car).
- E. Understands linear and angular momentum.
- F. Solves a variety of problems (e.g., collisions) using the conservation of linear and angular momentum.

Competency 030—The teacher understands the laws of thermodynamics.

The beginning teacher:

- A. Understands methods of heat transfer (i.e., convection, conduction, radiation).
- B. Understands the molecular interpretation of temperature and heat.
- C. Solves problems involving thermal expansion, heat capacity and the relationship between heat and other forms of energy.
- D. Applies the first law of thermodynamics to analyze energy transformations in a variety of everyday situations (e.g., electric light bulb, power-generating plant).
- E. Understands the concept of entropy and its relationship to the second law of thermodynamics.

Competency 031—The teacher understands the characteristics and behavior of waves.

The beginning teacher:

- A. Understands interrelationships among wave characteristics such as velocity, frequency, wavelength and amplitude and relates them to properties of sound and light (e.g., pitch, color).
- B. Compares and contrasts transverse and longitudinal waves.
- C. Describes how various waves are propagated through different media.
- D. Applies properties of reflection and refraction to analyze optical phenomena (e.g., mirrors, lenses, fiber-optic cable).
- E. Applies principles of wave interference to analyze wave phenomena, including acoustical (e.g., harmonics) and optical phenomena (e.g., patterns created by thin films and diffraction gratings).
- F. Identifies and interprets how wave characteristics and behaviors are used in medical, industrial and other real-world applications.

Competency 032—The teacher understands the fundamental concepts of quantum physics.

The beginning teacher:

- A. Interprets wave-particle duality.
- B. Identifies examples and consequences of the uncertainty principle.
- C. Understands the photoelectric effect.
- D. Understands the quantum model of the atom and can use it to describe and analyze absorption and emission spectra (e.g., line spectra, blackbody radiation) and other phenomenon (e.g., radioactive decay, nuclear forces, nuclear reactions).
- E. Explores real-world applications of quantum phenomena (e.g., lasers, photoelectric sensors, semiconductors, superconductivity).

Domain IX—Chemistry

Competency 033—The teacher understands the characteristics of matter and atomic structure.

The beginning teacher:

- A. Differentiates between physical and chemical properties and changes of matter.
- B. Explains the structure and properties of solids, liquids and gases.
- C. Identifies and analyzes properties of substances (i.e., elements and compounds) and mixtures.
- D. Models the atom in terms of protons, neutrons and electron clouds.
- E. Identifies elements and isotopes by atomic number and mass number and calculates average atomic mass of an element.

- F. Understands atomic orbitals and electron configurations and describes the relationship between electron energy levels and atomic structure.
- G. Understands the nature and historical significance of the periodic table.
- H. Applies the concept of periodicity to predict the physical (e.g., atomic and ionic radii) and chemical properties (e.g., electronegativity, ionization energy) of an element.

Competency 034—The teacher understands the properties of gases.

The beginning teacher:

- A. Understands interrelationships among temperature, moles, pressure and volume of gases contained within a closed system.
- B. Analyzes data obtained from investigations with gases in a closed system and determines whether the data are consistent with the ideal gas law.
- C. Applies the gas laws (e.g., Charles's law, Boyle's law, combined gas law) to describe and calculate gas properties in a variety of situations.
- D. Applies Dalton's law of partial pressure in various situations (e.g., collecting a gas over water).
- E. Understands the relationship between kinetic molecular theory and the ideal gas law.
- F. Knows how to apply the ideal gas law to analyze mass relationships between reactants and products in chemical reactions involving gases.

Competency 035—The teacher understands the properties and characteristics of ionic and covalent bonds.

The beginning teacher:

- A. Relates the electron configuration of an atom to its chemical reactivity.
- B. Compares and contrasts characteristics of ionic and covalent bonds.
- C. Applies the octet rule to construct Lewis structures.
- D. Identifies and describes the arrangement of atoms in molecules, ionic crystals, polymers and metallic substances.
- E. Understands the influence of bonding forces on the physical and chemical properties of ionic and covalent substances.
- F. Identifies and describes intermolecular and intramolecular forces.
- G. Uses intermolecular forces to explain the physical properties of a given substance (e.g., melting point, crystal structure).
- H. Applies the concepts of electronegativity, electron affinity and oxidation state to analyze chemical bonds.
 - I. Evaluates energy changes in the formation and dissociation of chemical bonds.
- J. Understands the relationship between chemical bonding and molecular geometry.

Competency 036—The teacher understands and interprets chemical equations and chemical reactions.

The beginning teacher:

- A. Identifies elements, common ions and compounds using scientific nomenclature.
- B. Uses and interprets symbols, formulas and equations in describing interactions of matter and energy in chemical reactions.
- C. Understands mass relationships involving percent composition, empirical formulas and molecular formulas.
- D. Interprets and balances chemical equations using conservation of mass and charge.
- E. Understands mass relationships in chemical equations and solves problems using calculations involving moles, limiting reagents and reaction yield.
- F. Identifies factors (e.g., temperature, pressure, concentration, catalysts) that influence the rate of a chemical reaction and describes their effects.
- G. Understands principles of chemical equilibrium and solves problems involving equilibrium constants.
- H. Identifies the chemical properties of a variety of common household chemicals (e.g., baking soda, bleach, ammonia) in order to predict the potential for chemical reactivity.

Competency 037—The teacher understands types and properties of solutions.

The beginning teacher:

- A. Analyzes factors that affect solubility (e.g., temperature, pressure, polarity of solvents and solutes) and rate of dissolution (e.g., surface area, agitation).
- B. Identifies characteristics of saturated, unsaturated and supersaturated solutions.
- C. Determines the molarity, molality, normality and percent composition of aqueous solutions.
- D. Analyzes precipitation reactions and derives net ionic equations.
- E. Understands the colligative properties of solutions (e.g., vapor pressure lowering, osmotic pressure changes, boiling-point elevation, freezing-point depression).
- F. Understands the properties of electrolytes and explains the relationship between concentration and electrical conductivity.
- G. Understands methods for measuring and comparing the rates of reaction in solutions of varying concentration.
- H. Analyzes models to explain the structural properties of water and evaluates the significance of water as a solvent in living organisms and the environment.

Competency 038—The teacher understands energy transformations that occur in physical and chemical processes.

The beginning teacher:

- A. Analyzes the energy transformations that occur in phase transitions.

- B. Solves problems in calorimetry (e.g., determining the specific heat of a substance, finding the standard enthalpy of formation and reaction of substances).
- C. Applies the law of conservation of energy to analyze and evaluate energy exchanges that occur in exothermic and endothermic reactions.
- D. Understands thermodynamic relationships among spontaneous reactions, entropy, enthalpy, temperature and Gibbs free energy.

Competency 039—The teacher understands nuclear fission, nuclear fusion and nuclear reactions.

The beginning teacher:

- A. Uses models to explain radioactivity and radioactive decay (i.e., alpha, beta and gamma).
- B. Interprets and balances equations for nuclear reactions.
- C. Compares and contrasts fission and fusion reactions (e.g., relative energy released in the reactions, mass distribution of products).
- D. Knows how to use the half-life of radioactive elements to solve real-world problems (e.g., carbon dating, radioactive tracers).
- E. Understands stable and unstable isotopes.
- F. Knows various issues associated with using nuclear energy (e.g., medical, commercial, environmental).

Competency 040—The teacher understands oxidation and reduction reactions.

The beginning teacher:

- A. Determines the oxidation state of ions and atoms in compounds.
- B. Identifies and balances oxidation and reduction reactions.
- C. Uses reduction potentials to determine whether a redox reaction will occur spontaneously.
- D. Explains the operation and applications of electrochemical cells.
- E. Analyzes applications of oxidation and reduction reactions from everyday life (e.g., combustion, rusting, electroplating, batteries).

Competency 041—The teacher understands acids, bases and their reactions.

The beginning teacher:

- A. Identifies the general properties of, and relationships among, acids, bases and salts.
- B. Identifies acids and bases using models of Arrhenius, Brønsted-Lowry and Lewis.
- C. Differentiates between strong and weak acids and bases.
- D. Applies the relationship between hydronium ion concentration and pH for acids and bases.

- E. Understands and analyzes acid-base equilibria and buffers.
- F. Analyzes and applies the principles of acid-base titration.
- G. Analyzes neutralization reactions based on the principles of solution concentration and stoichiometry.
- H. Describes the effects of acids and bases in the real world (e.g., acid precipitation, physiological buffering).

Domain X—Scientific Learning, Instruction and Assessment

Competency 042—The teacher understands research-based theoretical and practical knowledge about teaching science, how students learn science and the role of scientific inquiry in science instruction.

The beginning teacher:

- A. Knows research-based theories about how students develop scientific understanding and how developmental characteristics, prior knowledge, experience and attitudes of students influence science learning.
- B. Understands the importance of respecting student diversity by planning activities that are inclusive and selecting and adapting science curricula, content, instructional materials and activities to meet the interests, knowledge, understanding, abilities, possible career paths and experiences of all students, including English-language learners.
- C. Knows how to plan and implement strategies to encourage student self-motivation and engagement in their own learning (e.g., linking inquiry-based investigations to students' prior knowledge, focusing inquiry-based instruction on issues relevant to students, developing instructional materials using situations from students' daily lives, fostering collaboration among students).
- D. Knows how to use a variety of instructional strategies to ensure all students comprehend content-related texts, including how to locate, retrieve and retain information from a range of texts and technologies.
- E. Understands the science teacher's role in developing the total school program by planning and implementing science instruction that incorporates schoolwide objectives and the statewide curriculum as defined in the Texas Essential Knowledge and Skills (TEKS).
- F. Knows how to design and manage the learning environment (e.g., individual, small-group, whole-class settings) to focus and support student inquiries and to provide the time, space and resources for all students to participate in field, laboratory, experimental and nonexperimental scientific investigation.
- G. Understands the rationale for using active learning and inquiry methods in science instruction and how to model scientific attitudes such as curiosity, openness to new ideas and skepticism.
- H. Knows principles and procedures for designing and conducting an inquiry-based scientific investigation (e.g., making observations; generating questions; researching and reviewing current knowledge in light of existing evidence; choosing tools to gather and analyze evidence; proposing answers, explanations and predictions; and communicating and defending results).
- I. Knows how to assist students with generating, refining, focusing and testing scientific questions and hypotheses.

- J. Knows strategies for assisting students in learning to identify, refine and focus scientific ideas and questions guiding an inquiry-based scientific investigation; to develop, analyze and evaluate different explanations for a given scientific result; and to identify potential sources of error in an inquiry-based scientific investigation.
- K. Understands how to implement inquiry strategies designed to promote the use of higher-level thinking skills, logical reasoning and scientific problem solving in order to move students from concrete to more abstract understanding.
- L. Knows how to guide students in making systematic observations and measurements.
- M. Knows how to sequence learning activities in a way that uncovers common misconceptions, allows students to build upon their prior knowledge and challenges them to expand their understanding of science.

Competency 043—The teacher knows how to monitor and assess science learning in laboratory, field and classroom settings.

The beginning teacher:

- A. Knows how to use formal and informal assessments of student performance and products (e.g., projects, laboratory and field journals, rubrics, portfolios, student profiles, checklists) to evaluate student participation in and understanding of inquiry-based scientific investigations.
- B. Understands the relationship between assessment and instruction in the science curriculum (e.g., designing assessments to match learning objectives, using assessment results to inform instructional practice).
- C. Knows the importance of monitoring and assessing students' understanding of science concepts and skills on an ongoing basis by using a variety of appropriate assessment methods (e.g., performance assessment, self-assessment, peer assessment, formal/informal assessment).
- D. Understands the purposes, characteristics and uses of various types of assessment in science, including formative and summative assessments, and the importance of limiting the use of an assessment to its intended purpose.
- E. Understands strategies for assessing students' prior knowledge and misconceptions about science and how to use those assessments to develop effective ways to address the misconceptions.
- F. Understands characteristics of assessments, such as reliability, validity and the absence of bias in order to evaluate assessment instruments and their results.
- G. Understands the role of assessment as a learning experience for students and strategies for engaging students in meaningful self-assessment.
- H. Recognizes the importance of selecting assessment instruments and methods that provide all students with adequate opportunities to demonstrate their achievements.
- I. Recognizes the importance of clarifying teacher expectations by sharing evaluation criteria and assessment results with students.

Domain XI—The Engineering Method

Competency 044—The teacher has a working knowledge of engineering fundamentals.

The beginning teacher:

- A. Applies principles related to statics (e.g., moment, stress, strain) to analyze systems and solve problems.
- B. Applies principles of dynamics (e.g., force, acceleration, moment of inertia) to model and solve problems.
- C. Understands terminology (e.g., analog, digital) and concepts related to electric circuits (e.g., circuit analysis, digital logic circuits).
- D. Applies principles of fluid mechanics (e.g., Pascal's law, Bernoulli's law) to solve problems in fluid flow.
- E. Understands the applications of thermodynamics (e.g., heat transfer, energy conversions, efficiency) to engineering systems.
- F. Understands terminology and concepts related to control systems (e.g., input, output, feedback).
- G. Understands and applies the concepts of sketching and skills associated with computer-aided drafting and design.
- H. Applies mathematical principles of pneumatic pressure and flow to model and solve problems.
 - I. Applies mathematical principles of manufacturing processes in lathe operations and computer numerical control mill programming to model and solve problems.
- J. Applies mathematical principles of material engineering to model and solve problems.
- K. Applies mathematical principles for mechanical drives to model and solve problems.
- L. Applies mathematical principles of quality assurance (e.g., using precision measurement tools) to model and solve problems.
- M. Applies mathematical principles of robotics and computer programming of robotic mechanisms to model and solve problems.

Competency 045—The teacher understands the roles of mathematics, science and economics in the design process.

The beginning teacher:

- A. Solves problems using dimensional analysis and conversion factors.
- B. Understands methods of engineering estimation and approximation (e.g., error analysis).
- C. Applies knowledge of a variety of mathematical topics (e.g., trigonometry, vectors, matrices, calculus, Boolean algebra, binary number systems) to solve engineering problems.
- D. Relates principles of scientific inquiry to engineering design.
- E. Solves problems in engineering economics (e.g., simple and compound interest, depreciation, cost estimation, budgets, the time value of money).
- F. Integrates engineering, mathematics and physical science to solve engineering problems.

Competency 046—The teacher understands basic principles of information technology and computers and the role of information technology.

The beginning teacher:

- A. Understands terminology and concepts related to information technology (e.g., operating systems, networks, data transfer).
- B. Understands terminology and concepts related to programming (e.g., data structures, control loops, objects).
- C. Utilizes computer-based design and simulation tools in the design process.
- D. Analyzes problems using a variety of computer applications (e.g., spreadsheets, databases, mathematics packages).
- E. Communicates information using a variety of computer applications (e.g., graphics software, word processing software, presentation software).
- F. Understands principles of computer-integrated technologies.
- G. Understands the concepts of design processes for multiview computer-aided drafting and design drawings (e.g., for facilities layouts, precision part design, process design, computer-aided manufacturing for lathe, injection-mold design).

Competency 047—The teacher understands the engineering design process.

The beginning teacher:

- A. Understands the iterative design process.
- B. Knows how to formulate a problem so that it may be solved using engineering concepts.
- C. Identifies realistic constraints (e.g., safety features, costs, environmental impact, available resources) associated with an engineering problem.
- D. Knows how to collect, record, organize, analyze and communicate information needed to design a product, system or service.
- E. Understands the process of generating multiple solutions and applies decision-making skills for selecting optimal solutions.
- F. Applies oral, written and visual skills to communicate effectively with others (e.g., professional engineers, customers involved in the design process).
- G. Understands how to test a design solution using appropriate technology and how to redesign a product, system or service based on feedback and analysis of results.
- H. Understands the different techniques that engineering fields use to conceptualize and communicate ideas and concepts (e.g., sketching, schematics, working drawings, flow diagrams).

Competency 048—The teacher understands and applies knowledge of tools, equipment, materials and processes used in the design and production of prototypes.

The beginning teacher:

- A. Knows types of design tools, instrumentation and electrical measuring instruments used in engineering.
- B. Understands principles of product development (e.g., design, material selection, prototype construction, product testing).
- C. Understands a variety of manufacturing processes.
- D. Knows the fundamentals of quality assurance and procedures for evaluating a product (e.g., statistical tools).

Domain XII—The Engineering Profession

Competency 049—The teacher understands engineering and technology in a variety of contexts.

The beginning teacher:

- A. Understands the societal, cultural, economic, environmental and political contexts of engineering and technology.
- B. Understands how engineering and technology influence global society.
- C. Explains how societal and environmental needs, values, beliefs and institutions influence the design and development of engineering products, systems and services.
- D. Knows ways in which engineering and technology have influenced history.
- E. Understands the need for continuing education in the profession.

Competency 050—The teacher understands the concept of teaming, demonstrates knowledge of careers in engineering and understands the legal and ethical requirements of the engineering profession.

The beginning teacher:

- A. Knows and understands ethical standards, codes and certifications for the engineering profession and analyzes scenarios involving ethical issues that arise in engineering.
- B. Understands fundamental legal and ethical issues associated with patent, trademark, copyright and proprietary information.
- C. Knows and understands the various career fields in engineering (e.g., mechanical engineering, electrical engineering, civil engineering and biotechnology).
- D. Demonstrates knowledge of effective management skills (e.g., collaboration, resourcefulness, flexibility, delegation, supervision, professional appearance, verbal and nonverbal skills to enhance communication) and decision-making procedures.
- E. Understands the structure and function of multidisciplinary teams and strategies for working effectively within one, including effective written and oral communication skills.

- F. Analyzes ethics-related questions and scenarios that arise in engineering.
- G. Understands how to create and update a professional portfolio.

Competency 051—The teacher knows how to provide a safe and productive learning environment.

The beginning teacher:


- A. Understands safety procedures for various types of instructional activities (e.g., laboratory projects, field activities, classroom demonstrations).
- B. Knows how to access information related to the installation, maintenance and repair of equipment used in education facilities.
- C. Understands the safe and effective use of appropriate tools, technologies, materials and equipment.
- D. Knows regulations and guidelines (e.g., space requirements, environmental controls, safety equipment) for engineering education facilities and characteristics and layouts of effective instructional facilities used for engineering programs.
- E. Knows how to select, procure and use tools, equipment and materials (e.g., computer hardware and software, measuring tools, power tools) used in engineering education programs.

Competency 052—The teacher understands the importance of professional development and how to apply engineering knowledge to plan, implement and assess student learning.

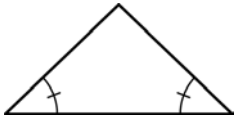

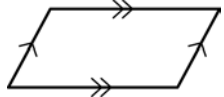
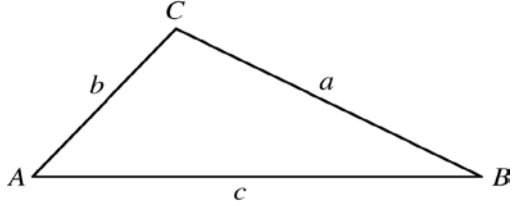
The beginning teacher:

- A. Develops a variety of instructional activities and design tasks in individual, small-group and large-group settings to guide students in learning engineering knowledge and skills.
- B. Uses open-ended, project-based activities to engage students in the learning process.
- C. Understands the importance of participating in professional activities related to engineering education.
- D. Understands the importance of participating in school and community efforts to promote the understanding of engineering and technology programs in school.
- E. Knows strategies for providing students with exposure to the engineering profession through student leadership development organizations, internships and work experiences.
- F. Knows how to use a variety of resources to enhance instruction and assessment.
- G. Knows how to effectively utilize laboratory and field experience to facilitate learning.
- H. Knows and understands the relationship between instruction and assessment.
- I. Knows state and national standards related to engineering education.

This reference material will also be available to you during the exam. To access it, click on the


 **Reference Materials** icon located in the lower-left corner of the screen.

Definitions and Formulas

<p style="text-align: center;">CALCULUS</p> <p>First Derivative: $f'(x) = \frac{dy}{dx}$</p> <p>Second Derivative: $f''(x) = \frac{d^2y}{dx^2}$</p> <p style="text-align: center;">PROBABILITY</p> <p>$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$</p> <p>$P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$</p>	<p style="text-align: center;">ALGEBRA</p> <p>$i^2 = -1$</p> <p>A^{-1} inverse of matrix A</p> <p>$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest, where A is the final value P is the principal r is the interest rate t is the term n is the number of divisions within the term</p> <p>$[x] = n$ Greatest integer function, where n is the integer such that $n \leq x < n + 1$</p>
<p style="text-align: center;">GEOMETRY</p> <p style="text-align: center;">Congruent Angles</p>  <p style="text-align: center;">Congruent Sides</p>  <p style="text-align: center;">Parallel Sides</p>  <p style="text-align: center;">Circumference of a Circle</p> <p style="text-align: center;">$C = 2\pi r$</p>	<p style="text-align: center;">VOLUME</p> <p>Cylinder: (area of base) \times height</p> <p>Cone: $\frac{1}{3}$ (area of base) \times height</p> <p>Sphere: $\frac{4}{3}\pi r^3$</p> <p>Prism: (area of base) \times height</p> <p style="text-align: center;">AREA</p> <p>Triangle: $\frac{1}{2}$ (base \times height)</p> <p>Rhombus: $\frac{1}{2}$ (diagonal₁ \times diagonal₂)</p> <p>Trapezoid: $\frac{1}{2}$ height (base₁ + base₂)</p> <p>Sphere: $4\pi r^2$</p> <p>Circle: πr^2</p> <p>Lateral surface area of cylinder: $2\pi rh$</p>
<p style="text-align: center;">TRIGONOMETRY</p> <p>Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$</p> <p>$c^2 = a^2 + b^2 - 2ab \cos C$</p> <p>Law of Cosines: $b^2 = a^2 + c^2 - 2ac \cos B$</p> <p>$a^2 = b^2 + c^2 - 2bc \cos A$</p>	

End of Definitions and Formulas

This reference material will also be available to you during the exam. To access it, click on the

 icon located in the lower-left corner of the screen.

Definitions and Physical Constants

The value of 9.8 m/s^2 is used for the acceleration of gravity near Earth's surface.

The universal gas constant is 8.314 J/K-mol or $0.08206 \text{ L-atm/K-mol}$.


Planck's constant is $6.6256 \times 10^{-34} \text{ J-s}$.

Avogadro's number is 6.022×10^{23} .

The right-hand rule is used with conventional current (the flow of positive charge from the positive terminal to the negative terminal).

End of Definitions and Physical Constants

This reference material will also be available to you during the exam. To access it, click on the

 icon located in the lower-left corner of the screen.

PERIODIC TABLE OF THE ELEMENTS

1 1A												18 8A					
1 H 1.01	2 2A											13 3A	14 4A	15 5A	16 6A	17 7A	2 He 4.00
3 Li 6.94	4 Be 9.01											5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00	9 F 19.00	10 Ne 20.18
11 Na 23.0	12 Mg 24.3	3 3B	4 4B	5 5B	6 6B	7 7B	8 8B	9 8B	10 8B	11 1B	12 2B	13 Al 27.0	14 Si 28.1	15 P 31.0	16 S 32.1	17 Cl 35.5	18 Ar 39.9
19 K 39.1	20 Ca 40.1	21 Sc 45.0	22 Ti 47.9	23 V 50.9	24 Cr 52.0	25 Mn 54.9	26 Fe 55.8	27 Co 58.9	28 Ni 58.7	29 Cu 63.5	30 Zn 65.4	31 Ga 69.7	32 Ge 72.6	33 As 74.9	34 Se 79.0	35 Br 79.9	36 Kr 83.8
37 Rb 85.5	38 Sr 87.6	39 Y 88.9	40 Zr 91.2	41 Nb 92.9	42 Mo 95.9	43 Tc (98.9)	44 Ru 101.1	45 Rh 102.9	46 Pd 106.4	47 Ag 107.9	48 Cd 112.4	49 In 114.8	50 Sn 118.7	51 Sb 121.8	52 Te 127.6	53 I 126.9	54 Xe 131.3
55 Cs 132.9	56 Ba 137.3	57-71	72 Hf 178.5	73 Ta 180.9	74 W 183.9	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.1	79 Au 197.0	80 Hg 200.6	81 Tl 204.4	82 Pb 207.2	83 Bi 209.0	84 Po (209)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra (226)	89-103	104 Rf (261)	105 Db (262)	106 Sg (266)	107 Bh (264)	108 Hs (277)	109 Mt (268)	110 Ds (271)	111 Rg (282)	112 Cn (285)	113 Nh (286)	114 Fl (289)	115 Mc (289)	116 Lv (293)	117 Ts (294)	118 Og (294)

Lanthanide Series	57 La 138.9	58 Ce 140.1	59 Pr 140.9	60 Nd 144.2	61 Pm (145)	62 Sm 150.4	63 Eu 152.0	64 Gd 157.3	65 Tb 158.9	66 Dy 162.5	67 Ho 164.9	68 Er 167.3	69 Tm 168.9	70 Yb 173.0	71 Lu 175.0
Actinide Series	89 Ac (227)	90 Th 232.0	91 Pa 231.0	92 U 238.0	93 Np (237)	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (262)

Preparation Manual

Section 4: Sample Selected-Response Questions Mathematics/Physical Science/Engineering 6–12 (274)

This section presents some sample exam questions for you to review as part of your preparation for the exam. To demonstrate how each competency may be assessed, sample questions are accompanied by the competency that they measure. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements do not appear on the actual exam.

For each sample exam question, there is a correct answer and a rationale for each answer option. The sample questions are included to illustrate the formats and types of questions you will see on the exam; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual exam.

The following reference materials will be available to you during the exam:

- Definitions and Formulas (see page 31)
- Definitions and Physical Constants (see page 32)
- Periodic Table (see page 33)

Domain I—Number Concepts

Competency 001—The teacher understands the real number system and its structure, operations, algorithms and representations.

1. If $S = \{-i, -1, 0, 1, i\}$, where $i^2 = -1$, which of the following statements is true?

- A. S is closed under addition.
- B. S is closed under subtraction.
- C. S is closed under multiplication.
- D. S is closed under division.

Answer _____

Competency 002—The teacher understands the complex number system and its structure, operations, algorithms and representations.

2. Let $z = a + bi$ where a and b are real numbers and $i^2 = -1$. Then $z^3 =$

- A. $(a^3 + ab^2) + i(a^2b + b^3)$
- B. $(a^3 + 3ab^2) + i(3a^2b + b^3)$
- C. $(a^3 - ab^2) + i(a^2b - b^3)$
- D. $(a^3 - 3ab^2) + i(3a^2b - b^3)$

Answer _____

Domain II—Patterns and Algebra

Competency 005—The teacher understands attributes of functions, relations and their graphs.

Use the functions below to answer the question that follows.

$$f(x) = 2x^2$$

$$g(x) = 4x + x^2$$

3. Which of the following is $f(g(x))$?

- A. $8x^2 + 4x^4$
- B. $4x + 3x^2$
- C. $32x^2 + 16x^3 + 2x^4$
- D. $8x^3 + 2x^4$

Answer _____

4. If f is a function defined by $f(x) = -\frac{1}{3}x + 1$, for all x , which of the following is f^{-1} ?

- A. $f^{-1}(x) = \frac{1}{3}x + 1$
- B. $f^{-1}(x) = -3x + 3$
- C. $f^{-1}(x) = 3x - 3$
- D. $f^{-1}(x) = -\frac{1}{3}x + 1$

Answer _____

Competency 007—The teacher understands polynomial, rational, radical, absolute value and piecewise functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

Use the equation below to answer the question that follows.

$$f(x) = x^3 + 27$$

5. Which of the following statements is true?

- A. $f(x)$ has no real roots.
- B. $f(x)$ has one real root and two complex roots.
- C. $f(x)$ has two real roots and one complex root.
- D. $f(x)$ has three real roots.

Answer _____

6. Let f be the function defined by $f(x) = 5 + \sqrt{1 - (x - 2)^2}$ for all real numbers x for which $f(x)$ is a real number. Which of the following is the domain of f ?

- A. $[0, 1]$
- B. $[1, 3]$
- C. $[1, \infty)$
- D. $(-\infty, 3]$

Answer _____

Competency 009—The teacher understands trigonometric and circular functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

Use the problem below to answer the question that follows.

Let $f(x) = \cos x - \sin x$. What are all solutions to $f(x) = 0$?

7. Which of the following is true?

- A. $f(x)$ has no solutions.
- B. $f(x)$ has exactly one solution, $x = \frac{\pi}{4}$.
- C. $f(x)$ has an infinite number of solutions, $x = \frac{\pi}{2} \pm 2n\pi$, where n is any integer.
- D. $f(x)$ has an infinite number of solutions, $x = \frac{\pi}{4} \pm n\pi$, where n is any integer.

Answer _____

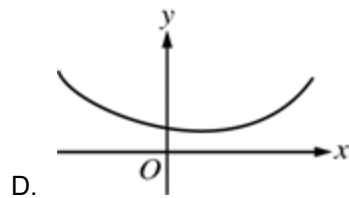
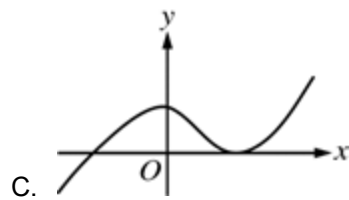
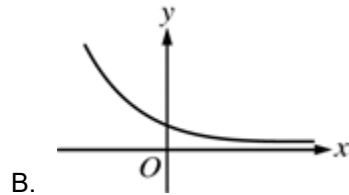
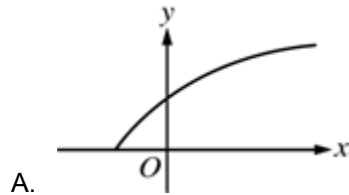
Competency 010—The teacher understands and solves problems using differential and integral calculus.

8. Which of the following represents an expression that will calculate the area enclosed by the graph of $y = -2x + 5$ and the positive x - and y -axes?

- A. $\int_0^5 (-2x + 5) dx$
- B. $\int_{-2}^5 (-2x + 5) dx$
- C. $\int_0^{5/2} (-2x + 5) dx$
- D. $\int_{5/2}^5 (-2x + 5) dx$

Answer _____

9. If f is a differentiable real-valued function such that $f'(x) > 0$ and $f''(x) < 0$ for all $x \geq 0$, which of the following could be the graph of f in the xy -plane?



Answer _____

Domain III—Geometry and Measurement

Competency 012—The teacher understands geometries, in particular Euclidian geometry, as axiomatic systems.

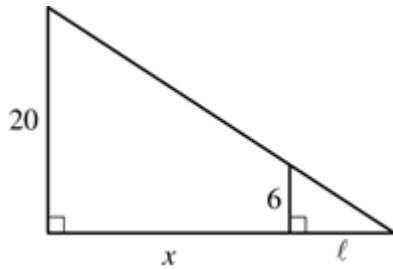
10. In Euclidean geometry, the sum of the interior angles of a triangle is 180° . Let ABC be a triangle in elliptical geometry. Which of the following statements about the measures of angles A , B and C is true?

- A. $m\angle A + m\angle B + m\angle C = 180^\circ$
- B. $m\angle A + m\angle B + m\angle C < 180^\circ$
- C. $m\angle A + m\angle B + m\angle C > 180^\circ$
- D. $m\angle A + m\angle B + m\angle C = \infty$

Answer _____

Competency 013—The teacher understands the results, uses and applications of Euclidian geometry.

Use the figure below to answer the question that follows.



11. A student drew the figure shown to model a person 6 feet tall walking away from a lamppost 20 feet tall. Based on the figure, which of the following is true?

- A. $\frac{l}{6} = \frac{x+l}{20}$
- B. $\frac{l}{6} = \frac{x}{20}$
- C. $\frac{x+l}{6} = \frac{x}{20}$
- D. $\frac{x+l}{6} = \frac{l}{20}$

Answer _____

Competency 014—The teacher understands coordinate, transformational and vector geometry and their connections.

12. Let $v = (a_1, b_1, c_1)$ and $w = (a_2, b_2, c_2)$ be vectors in R^3 . If the dot product of two vectors is given by $v \cdot w = a_1a_2 + b_1b_2 + c_1c_2$, and $\|v\| = \sqrt{v \cdot v}$, then which of the following is equivalent to $(v + w) \cdot v - (v + w) \cdot w = 0$?

- A. $\|v\|^2 + v \cdot w + \|w\|^2 = 0$
- B. $\|v\|^2 + 2v \cdot w - \|w\|^2 = 0$
- C. $\|v\|^2 - 2v \cdot w + \|w\|^2 = 0$
- D. $\|v\|^2 - \|w\|^2 = 0$

Answer _____

Domain IV—Probability and Statistics

Competency 015—The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.

13. A calculus teacher gave an exam to 83 students where the highest possible score was 100. The teacher calculated that the average (arithmetic mean) score was 72.4, the median score was 72.0, the mode of the scores was 68.0 and the standard deviation of the scores was 12.0. There was a makeup exam for one student, and the student's score was

100. The teacher recalculated the average, median, mode, and standard deviation for the scores of all 84 students. Which of the following statements is true?

- A. The average and the standard deviation increased.
- B. The average, median and mode decreased.
- C. The median, mode and standard deviation decreased.
- D. The average, median and mode where unchanged.

Answer _____

Competency 016—The teacher understands concepts and applications of probability.

14. A cube has its six faces colored red, white, blue, green, yellow and violet. Each of the six faces is equally likely to be face up when the cube is tossed. The cube is tossed twice. What is the probability that the red face will appear face up on the first toss, but will not appear face up on the second toss?

- A. $\frac{1}{36}$
- B. $\frac{5}{36}$
- C. $\frac{5}{12}$
- D. 1

Answer _____

Domain V—Mathematical Processes and Perspectives

Competency 018—The teacher understands mathematical reasoning and problem solving.

15. In 2012, the music industry in Country Y generated revenues of 900 million dollars in sound recordings. Jazz and blues sound recordings accounted for approximately 4.7 percent of the country's sale of sound recordings. The revenue from the sale of jazz and blues recordings in Country Y increased by 12.0 percent from 2010 to 2012. What was the approximate revenue from the sale of jazz and blues recordings in Country Y in 2010?

- A. 38 million dollars
- B. 60 million dollars
- C. 450 million dollars
- D. 803 million dollars

Answer _____

Use the equation below to answer the question that follows.

$$\cos(A + B) = \cos A + \cos B$$

16. A teacher gives his class the trigonometric equation above and asks the students to verify whether the equation is

valid for all angles A and B . If the equation is not true for all angles A and B , he asks his students to give a counterexample that shows the equation to be false. If the equation is true, for all angles A and B , the students are asked to give a proof. By introducing the concept in this manner, the teacher

- A. understands the problem solving process.
- B. evaluates how well a mathematical model represents a real-world situation.
- C. recognizes and uses multiple representations of a mathematical concept.
- D. applies correct mathematical reasoning to make conjectures, and uses deductive methods to evaluate the validity of conjectures.

Answer _____

Domain VII—Scientific Inquiry and Processes

Competency 022—The teacher understands how to select and manage learning activities to ensure the safety of all students and the correct use and care of organisms, natural resources, materials, equipment and technologies.

17. During a laboratory investigation on density, the side of an aluminum cube is measured to be 2.49 cm. The volume of the cube should be recorded with the correct number of significant figures as which of the following?

- A. 15 cm^3
- B. 15.4 cm^3
- C. 15.44 cm^3
- D. 15.438 cm^3

Answer _____

Competency 023—The teacher understands the nature of science, the process of scientific inquiry and the unifying concepts that are common to all sciences.

18. Which of the following is a scientific inference?

- A. A mass scale is accurate to 0.01 grams.
- B. Two charged spheres are observed to attract each other.
- C. The periodic wobbling of a distant star suggests that a planet is in orbit around it.
- D. The average acceleration of a cart can be determined from the change in its velocity over a given time interval.

Answer _____

Domain VIII—Physics

Competency 025—The teacher understands the description of motion in one and two dimensions.

19. Vector quantities include which of the following? Select all that apply.

- A. Force
- B. Kinetic energy
- C. Acceleration
- D. Electric field

Answer _____

Competency 027—The teacher understands the concepts of gravitational and electromagnetic forces in nature.

20. On the basis of Coulomb's law, which of the following is true about the electrostatic force that two charged objects X and Y exert on one another?

- A. The force is directly proportional to the sum of the squares of the charges of the two objects.
- B. The force is inversely proportional to the square of the distance between the two objects.
- C. If the charges on objects X and Y are of the same sign, then the force on object X is in the same direction as the force on object Y .
- D. If the charges on objects X and Y have opposite signs, then the force on object X is in the same direction as the force on object Y .

Answer _____

Competency 028—The teacher understands applications of electricity and magnetism.

21. Two $6\ \Omega$ resistors are connected in parallel to an ideal $24\ \text{V}$ source. What is the current through the voltage source?

- A. $2\ \text{A}$
- B. $4\ \text{A}$
- C. $6\ \text{A}$
- D. $8\ \text{A}$

Answer _____

Competency 029—The teacher understands the conservation of energy and momentum.

22. In an isolated system, which of the following quantities are conserved? Select all that apply.

- A. Velocity
- B. Linear momentum
- C. Angular momentum
- D. Energy

Answer _____

Competency 031—The teacher understands the characteristics and behavior of waves.

23. Which of the following is a consequence of refraction of light?

- A. The formation of an image after light passes through a lens
- B. The bending and spreading of light as it passes through a narrow slit
- C. The pattern formed when light passes through two narrow, parallel slits
- D. The focusing of light into a small area after reflecting from a curved mirror

Answer _____

Competency 032—The teacher understands the fundamental concepts of quantum physics.

24. According to the Bohr model of the hydrogen atom, what happens when an electron makes a transition from a lower energy level to a higher level?

- A. The nucleus loses kinetic energy.
- B. A photon is absorbed by the atom.
- C. A neutron decays into a proton, an electron, and a neutrino.
- D. A positron makes a transition from the higher level to the lower level.

Answer _____

Domain IX—Chemistry

Competency 033—The teacher understands the characteristics of matter and atomic structure.

25. Of the following species, which has the largest ionic radius?

- A. Na^+
- B. Mg^{2+}
- C. Al^{3+}
- D. Cl^-

Answer _____

Competency 035—The teacher understands the properties and characteristics of ionic and covalent bonds.

26. Which of the following best describes the molecular geometry of formaldehyde H_2CO ?

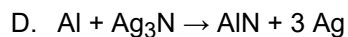
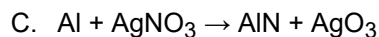
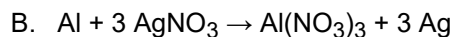
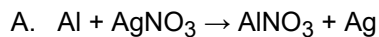
- A. Tetrahedral
- B. Trigonal planar
- C. Trigonal pyramidal

D. T-shaped

Answer _____

Competency 036—The teacher understands and interprets chemical equations and chemical reactions.

27. Which of the following is the balanced equation for the displacement reaction of aluminum with silver nitrate?



Answer _____

Competency 037—The teacher understands types and properties of solutions.

28. Of the following solutions, which has the highest electrical conductivity?

A. 1 M NaCl

B. 1 M HCH_3CO_2

C. 1 M HCl

D. 1 M Na_2SO_4

Answer _____

Competency 038—The teacher understands energy transformations that occur in physical and chemical processes.

Use the table below to answer the question that follows.

Specific heat of ice	2.03 J/g·°C
Specific heat of water	4.18 J/g·°C
Heat of fusion of ice	334 J/g

29. Based on the data in the table above, what is the change in heat content of 100. g of water when it is cooled from 20.0°C to -20°C?

A. -45,820 J

B. -12,420 J

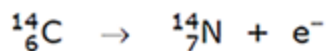
C. +33,400 J

D. +45,820 J

Answer _____

Competency 039—The teacher understands nuclear fission, nuclear fusion and nuclear reactions.

Use the equation below to answer the question that follows.



30. The isotope carbon-14 decays according to the equation shown above. The transformation is an example of

- A. alpha decay.
- B. beta decay.
- C. gamma decay.
- D. x-ray emission.

Answer _____

Competency 040—The teacher understands oxidation and reduction reactions.

Use the table below to answer the question that follows.

Reduction Half-Reaction	E° (V)
$\text{Ag}^{+}(\text{aq}) + e^{-} \rightarrow \text{Ag}(\text{s})$	+0.80
$\text{Cu}^{2+}(\text{aq}) + 2e^{-} \rightarrow \text{Cu}(\text{s})$	+0.34
$\text{Fe}^{2+}(\text{aq}) + 2e^{-} \rightarrow \text{Fe}(\text{s})$	-0.44
$\text{Zn}^{2+}(\text{aq}) + 2e^{-} \rightarrow \text{Zn}(\text{s})$	-0.76

31. Based on the standard reduction potentials in the table above, which of the following reactions will occur spontaneously?

- A. $2 \text{Ag}(\text{s}) + \text{Fe}^{2+}(\text{aq}) \rightarrow 2 \text{Ag}^{+}(\text{aq}) + \text{Fe}(\text{s})$
- B. $\text{Cu}(\text{s}) + \text{Fe}^{2+}(\text{aq}) \rightarrow \text{Cu}^{2+}(\text{aq}) + \text{Fe}(\text{s})$
- C. $\text{Fe}(\text{s}) + \text{Zn}^{2+}(\text{aq}) \rightarrow \text{Fe}^{2+}(\text{aq}) + \text{Zn}(\text{s})$
- D. $\text{Zn}(\text{s}) + \text{Cu}^{2+}(\text{aq}) \rightarrow \text{Zn}^{2+}(\text{aq}) + \text{Cu}(\text{s})$

Answer _____

Domain X—Scientific Learning, Instruction and Assessment

Competency 042—The teacher understands research-based theoretical and practical knowledge about teaching science, how students learn science and the role of scientific inquiry in science instruction.

32. Which of the following is an example of inquiry-based science learning?

- A. A group of students uses small carts to study whether kinetic energy is conserved when they collide.
- B. A classroom of students watches a video on the similarities and distinctions between work and impulse.

- C. A teacher gives a presentation on the role heating and work play in the conservation of energy.
- D. A student conducts a literature search on the role of entropy in thermodynamic systems.

Answer _____

Domain XI—The Engineering Method

Competency 044—The teacher has a working knowledge of engineering fundamentals.

Use the figure below to answer the question that follows.

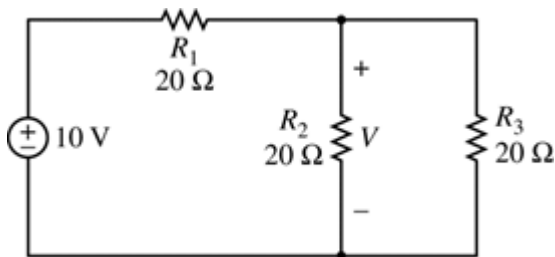


33. A technician designs a hydraulic system to raise heavy objects. The cross-sectional area of the smaller piston is A_1 and the cross-sectional area A_2 of the larger piston is 10 times that of the smaller piston. The system is filled with an incompressible fluid. If the mechanical advantage of the system is 10, how much force must be applied to lift a 200-kilogram mass and on which piston should the force be applied? (Assume that the gravitational acceleration is 10 meters per second squared.)

- A. A force of 200 newtons must be applied to the larger piston.
- B. A force of 200 newtons must be applied to the smaller piston.
- C. A force of 20 newtons must be applied to the larger piston.
- D. A force of 20 newtons must be applied to the smaller piston.

Answer _____

Use the closed electrical circuit below to answer the question that follows.



34. The closed electrical circuit shown consists of a generator and three resistors, R_1 , R_2 and R_3 . The electromotive force of the generator is 10 volts. Which of the following is the second resistor's voltage, V ?

- A. 3.33 V
- B. 5.00 V

- C. 6.67 V
- D. 10.00 V

Answer _____

35. A broken down car, with mass 1859 kilograms, is accelerated southward at 1.32 meters per second squared when pulled by a tow truck. Which of the following is closest to the force of friction if the tow truck exerts 2.84 kilonewtons, southward on the car?

- A. 0.386 kilonewtons
- B. 0.842 kilonewtons
- C. 5.293 kilonewtons
- D. 2840 kilonewtons

Answer _____

Competency 046—The teacher understands basic principles of information technology and computers and the role of information technology.

36. A wireless communication system has been set up between two locations and is said to be problematic because it is band-limited. Which of the following is the most likely cause of the problem?

- A. It takes too long to encode the bits that are to be transmitted
- B. There is not enough computer memory in the system
- C. There is not enough dedicated transmission bandwidth to send the information quickly enough
- D. There are types of transmissions that are forbidden due to security risks

Answer _____

Domain XII—The Engineering Profession

Competency 050—The teacher understands the concept of teaming, demonstrates knowledge of careers in engineering and understands the legal and ethical requirements of the engineering profession.

Use the table below to answer the question that follows.

Maximum repayment period	60 months
Maximum monthly payment	\$20,000
Maximum total repayment	\$1,000,000

37. The board of directors of a small engineering firm has approved a long-term investment in one of their projects that will require securing a \$500,000 loan. The guidelines for the loan are listed in the table shown. Which of the following options represents an offer from a commercial banking institution that meets the criteria set by the board?

- A. 48 months, at 12% annual interest, compounded quarterly

- B. 30 months, at 24% annual interest, compounded quarterly
- C. 36 months, at 3% simple and 36% annual interest
- D. 24 months, at 4% simple and 48% annual interest

Answer _____

38. An engineering manager who supervises a team of twelve people has been assigned to generate reports from all four sectors of production. The manager is newly appointed and has been asked by the boss to practice delegation. Which of the following actions are most appropriate and efficient to get the reports completed?

- A. Personally writing all the reports
- B. Meet with senior members of the team to determine which team members are best suited to write the reports, and then let the senior members assign the reports to whoever is appropriate. The new manager will review and edit the reports before sending to the boss.
- C. Assigning one report to each of the twelve team members and then personally reviewing, consolidating and rewriting all the reports to correct any important omissions
- D. Assigning all four reports to one randomly selected team member, assuming that all members are well qualified to write a report

Answer _____

39. An engineering firm has been hired by the city to construct a bridge over a newly formed irrigation channel. The main professional engineer (PE) is out on medical leave, and another engineer has taken his place as temporary PE. The temporary PE has just been recently certified in Civil Transportation by the National Council of Examiners for Engineering and Surveying (NCEES) and he is competent but unsure of himself in certain areas of the safety requirements for the bridge. He would prefer to delay signoff until the experienced PE comes back, but an official from the city asks the temporary PE if it is possible to speed up the sign-off process. By signing off without having the experienced PE review the plan, this NCEES-licensed PE will be violating his obligation to which of the following?

- A. To society
- B. To society, his employer and his clients
- C. To other licensees
- D. To society, his employer, his clients and other licensees

Answer _____

40. A chemical engineering company, ChemA, has developed a new type of chemical process and has patented a new piece of equipment to run the process. An engineer owns a small business that provides expendable parts and supplies for use with the equipment. According to the code of ethics, when asked at a press conference to give his opinion on the effectiveness of the new equipment over equipment using older techniques, the engineer should

- A. refrain from making any comments since he has the potential to profit from the acceptance of the new equipment by the users.
- B. make statements that are only highly technical in nature in order to confuse the press.

- C. disclose his relationship with the company that manufactures the processing equipment before commenting or giving an opinion.
- D. read a statement that has been prepared by ChemA specifically for the purposes of responding to questions from the press.

Answer _____

Competency 051—The teacher knows how to provide a safe and productive learning environment.

41. A good lab instructor promotes a policy of safety awareness and helps students, faculty and staff develop positive attitudes toward safety. Which of the following is the best procedure to follow to help students prevent accidents in the lab?

- A. Fully explaining all safety considerations to students at the beginning of each lab
- B. Demonstrating safety procedures and identifying potential hazards before students begin their work and identifying hazards and applying safety procedures as appropriate throughout the lab
- C. Posting and distributing written directions or warnings, when appropriate, about the safe handling of materials and equipment used in instruction
- D. Regularly administering safety quizzes to students to ensure that safe procedures are followed

Answer _____

42. A group of students are participating in a lab activity in which they design and test different types of electrical circuits. Which of the following safety procedures is most important for the students to follow?

- A. Removing drinks from the laboratory area prior to the activity
- B. Wearing safety goggles and aprons while in the lab area
- C. Disposing of chewing gum before starting the lab activity
- D. Keeping loose hair and clothing tied back during the lab activity

Answer _____

Use the tables below to answer the question that follows.

Device	Power Rating
Vacuum	1360 W
Spotlight	720 W

Device Amperage Rating	Power Rating
1–7 amps	16 gauge
8–10 amps	14 gauge
11–14 amps	12 gauge
14+ amps	10 gauge

43. A technician is setting up the machine shop lab and needs to use a spotlight and shop vacuum, with the requirements shown. If the lab has a 120V source, and the technician is plugging both appliances into the same extension cord in parallel, what is the minimum gauge wire the setup requires?

- A. 16 gauge
- B. 14 gauge
- C. 12 gauge
- D. 10 gauge

Answer _____

Competency 052—The teacher understands the importance of professional development and how to apply engineering knowledge to plan, implement and assess student learning.

44. A group of eighth grade students received the following instruction from their teacher:

"Using less than 15 feet of masking tape and 110 paper straws, design a bridge that will span the two tables in the classroom and hold the weight of the electric pencil sharpener. Test to determine if plastic straws are better for building the structure than paper straws. Then try wooden frozen-dessert sticks instead of straws."

Which of the following best describes this type of activity?

- A. An open-ended, project-based activity
- B. A focused, project-based activity
- C. A controlled experiment
- D. A qualitative inquiry

Answer _____

Preparation Manual

Section 4: Sample Selected-Response Response Answers and Rationales

Mathematics/Physical Science/Engineering 6–12 (274)

This section presents some sample exam questions for you to review as part of your preparation for the exam. To demonstrate how each competency may be assessed, sample questions are accompanied by the competency that they measure. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements do not appear on the actual exam.

For each sample exam question, there is a correct answer and a rationale for each answer option. The sample questions are included to illustrate the formats and types of questions you will see on the exam; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual exam.

The following reference materials will be available to you during the exam:

- Definitions and Formulas (see page 31)
- Definitions and Physical Constants (see page 32)
- Periodic Table (see page 33)

Domain I—Number Concepts

Competency 001—The teacher understands the real number system and its structure, operations, algorithms and representations.

1. If $S = \{-i, -1, 0, 1, i\}$, where $i^2 = -1$, which of the following statements is true?

- A. S is closed under addition.
- B. S is closed under subtraction.
- C. S is closed under multiplication.
- D. S is closed under division.

Answer

Option C is correct because the multiplication of any two elements of S returns an element of S . **Option A is incorrect** because $1 + 1 = 2$, and 2 is not an element of S . **Option B is incorrect** because $-1 - 1 = -2$, and -2 is not an element of S . **Option D is incorrect** because division by 0 is undefined.

Competency 002—The teacher understands the complex number system and its structure, operations, algorithms and representations.

2. Let $z = a + bi$ where a and b are real numbers and $i^2 = -1$. Then $z^3 =$

- A. $(a^3 + ab^2) + i(a^2b + b^3)$

B. $(a^3 + 3ab^2) + i(3a^2b + b^3)$

C. $(a^3 - ab^2) + i(a^2b - b^3)$

D. $(a^3 - 3ab^2) + i(3a^2b - b^3)$

Answer

Option D is correct because:

$$\begin{aligned} z^3 &= (a + bi)^3 \\ &= a^3 + 3a^2bi + 3a(bi)^2 + (bi)^3 \\ &= a^3 + 3a^2bi - 3ab^2 - ib^3 \\ &= (a^3 - 3ab^2) + i(3a^2b - b^3) \end{aligned}$$

Option A is incorrect because the coefficients of ab^2 and a^2b are incorrect. **Option B is incorrect** because the coefficients of ab^2 and b^3 are incorrect. **Option C is incorrect** because the coefficients of ab^2 , a^2b and b^3 are incorrect.

Domain II—Patterns and Algebra

Competency 005—The teacher understands attributes of functions, relations and their graphs.

Use the functions below to answer the question that follows.

$$f(x) = 2x^2$$

$$g(x) = 4x + x^2$$

3. Which of the following is $f(g(x))$?

A. $8x^2 + 4x^4$

B. $4x + 3x^2$

C. $32x^2 + 16x^3 + 2x^4$

D. $8x^3 + 2x^4$

Answer

Option C is correct because when evaluating the composite function, $g(x)$ is replaced by $4x + x^2$, then this expression is squared, and finally it is multiplied by 2, as indicated by $f(x)$. In other words:

$$\begin{aligned} f(g(x)) &= f(4x + x^2) \\ &= 2(4x + x^2)^2 \\ &= 2(16x^2 + 8x^3 + x^4) \\ &= 32x^2 + 16x^3 + 2x^4 \end{aligned}$$

Option A is incorrect. Option A is $g(f(x))$. **Option B is incorrect.** Option B is $f(x) + g(x)$. **Option D is incorrect.** Option D is $f(x)g(x)$.

4. If f is a function defined by $f(x) = -\frac{1}{3}x + 1$, for all x , which of the following is f^{-1} ?

A. $f^{-1}(x) = \frac{1}{3}x + 1$

B. $f^{-1}(x) = -3x + 3$

C. $f^{-1}(x) = 3x - 3$

D. $f^{-1}(x) = -\frac{1}{3}x + 1$

Answer

Option B is correct because to find the inverse function f^{-1} of the original function f , first substitute y for $f(x)$, in the equation, then switch the variables and solve for the new y , or $f^{-1}(x)$:

$$f(x) = -\frac{1}{3}x + 1$$

$$y = -\frac{1}{3}x + 1$$

$$x = -\frac{1}{3}y + 1$$

$$x - 1 = -\frac{1}{3}y$$

$$-3(x - 1) = y$$

$$f^{-1}(x) = -3x + 3$$

If $g(x) = f^{-1}(x)$, then $(f \circ g)(x) = x$.

Option A is incorrect because the composition of $g(x) = \frac{1}{3}x + 1$ with $f(x) = -\frac{1}{3}x + 1$ is $(f \circ g)(x) = -\frac{1}{3}\left(\frac{1}{3}x + 1\right) + 1 = -\frac{1}{9}x + \frac{2}{3} \neq x$.

Option C is incorrect because the composition of $g(x) = 3x - 3$ with $f(x) = -\frac{1}{3}x + 1$ is $(f \circ g)(x) = -\frac{1}{3}(3x - 3) + 1 = -x + 2 \neq x$.

Option D is incorrect because the composition of $g(x) = -\frac{1}{3}x + 1$ with $f(x) = -\frac{1}{3}x + 1$ is $(f \circ g)(x) = -\frac{1}{3}\left(-\frac{1}{3}x + 1\right) + 1 = \frac{1}{9}x + \frac{2}{3} \neq x$.

Competency 007—The teacher understands polynomial, rational, radical, absolute value and piecewise functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

Use the equation below to answer the question that follows.

$$f(x) = x^3 + 27$$

5. Which of the following statements is true?

- A. $f(x)$ has no real roots.
- B. $f(x)$ has one real root and two complex roots.
- C. $f(x)$ has two real roots and one complex root.
- D. $f(x)$ has three real roots.

Answer

Option B is correct because the expression in the function can be factored as the sum of two cubes. This means

$$\begin{aligned}f(x) &= x^3 + 27 \\ &= x^3 + 3^3 \\ &= (x + 3)(x^2 - 3x + 9)\end{aligned}$$

The first of these polynomials has a real root of -3 .

To determine the types of roots for the second polynomial, use the Discriminant of the quadratic equation

$$\begin{aligned}x^2 - 3x + 9, \\ b^2 - 4ac &= (-3)^2 - 4(1)(9) \\ &= 9 - 36 = -27\end{aligned}$$

The Discriminant of the second polynomial is negative, so it must have two complex roots. So $f(x)$ has one real root, $x = -3$, and two complex roots, $x = \frac{3 + 3i\sqrt{3}}{2}$ and $x = \frac{3 - 3i\sqrt{3}}{2}$.

Option A is incorrect because -3 is a real root of the function. **Option C is incorrect** because if complex roots of polynomials with real coefficients occur, they must occur in pairs. **Option D is incorrect** because there is only one real root, -3 .

6. Let f be the function defined by $f(x) = 5 + \sqrt{1 - (x - 2)^2}$ for all real numbers x for which $f(x)$ is a real number. Which of the following is the domain of f ?

- A. $[0, 1]$
- B. $[1, 3]$
- C. $[1, \infty)$
- D. $(-\infty, 3]$

Answer

Option B is correct because in order for $f(x)$ to be a real number, the radicand must be greater than or equal to 0:

$$\begin{aligned}
1 - (x - 2)^2 &\geq 0 \\
-(x - 2)^2 &\geq -1 \\
(x - 2)^2 &\leq 1 \\
(x - 2)^2 - 1 &\leq 0 \\
(x - 2 - 1)(x - 2 + 1) &\leq 0 \\
(x - 3)(x - 1) &\leq 0
\end{aligned}$$

This shows that the x -intercepts for the function are at $x = 1$ and $x = 3$. To determine in which intervals $(x - 1)(x - 3) < 0$, test values that are

- less than 1: let $x = 0$, $f(0) = (-1)(-3) = 3$
So on the interval $(-\infty, 1)$, $(x - 1)(x - 3) > 0$.
- between 1 and 3: Let $x = 2$, $f(2) = (2 - 1)(2 - 3) = -1$
So on the interval $(1, 3)$, $(x - 1)(x - 3) < 0$.
- greater than 3: $x = 4$, $f(4) = (4 - 1)(4 - 3) = 3$
On the interval $(3, \infty)$, $(x - 1)(x - 3) > 0$.

Therefore the domain of f , which satisfies the inequality $(x - 3)(x - 1) \leq 0$, is $[1, 3]$. **Option A is incorrect** because $f(0) = 5 + \sqrt{-3}$, which is not a real number. **Option C is incorrect** because $f(4) = 5 + \sqrt{-3}$, which is not a real number. **Option D is incorrect** because $f(0) = 5 + \sqrt{-3}$, which is not a real number.

Competency 009—The teacher understands trigonometric and circular functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.

Use the problem below to answer the question that follows.

Let $f(x) = \cos x - \sin x$. What are all solutions to $f(x) = 0$?

7. Which of the following is true?

- A. $f(x)$ has no solutions.
- B. $f(x)$ has exactly one solution, $x = \frac{\pi}{4}$.
- C. $f(x)$ has an infinite number of solutions, $x = \frac{\pi}{2} \pm 2n\pi$, where n is any integer.
- D. $f(x)$ has an infinite number of solutions, $x = \frac{\pi}{4} \pm n\pi$, where n is any integer.

Answer

Option D is correct because the difference of the functions of cosine and sine will produce an oscillating function that will intersect with the x -axis, periodically. Use the following steps to determine the zeros of the function:

$$\begin{aligned}\cos x - \sin x &= 0 \\ (\cos x - \sin x)^2 &= 0^2 \\ \cos^2 x - 2\sin x \cos x + \sin^2 x &= 0 \\ 1 - 2\sin x \cos x &= 0 \\ 2\sin x \cos x &= 1 \\ \sin 2x &= 1\end{aligned}$$

This means that, where n is an integer,

$$2x = \frac{\pi}{2} \pm 2n\pi$$

$$x = \frac{\pi}{4} \pm n\pi$$

Options A and B are incorrect because $f(x)$ has an infinite number of zeros. **Option C is incorrect** because when $n = 0$, $x = \frac{\pi}{2}$, which is not a solution of $f(x)$.

Competency 010—The teacher understands and solves problems using differential and integral calculus.

8. Which of the following represents an expression that will calculate the area enclosed by the graph of $y = -2x + 5$ and the positive x - and y -axes?

- A. $\int_0^5 (-2x + 5)dx$
- B. $\int_{-2}^5 (-2x + 5)dx$
- C. $\int_0^{5/2} (-2x + 5)dx$
- D. $\int_{5/2}^5 (-2x + 5)dx$

Answer

Option C is correct; to find the area under a curve, use the definite integral $\int_a^b f(x)dx$, where $f(x)$ is the function, a is the lower bound of the definite integral, and b is the upper bound. When the function $y = -2x + 5$ is graphed along with the positive x - and y -axes, a right triangle is formed. The lower bound is $x = 0$ (the leftmost x -value possible), and the upper bound is the x -intercept of $f(x)$. To find the upper bound, let $y = 0$ and solve for x :

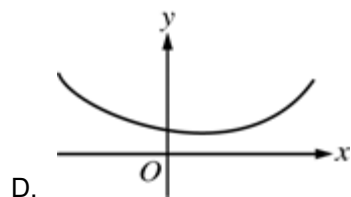
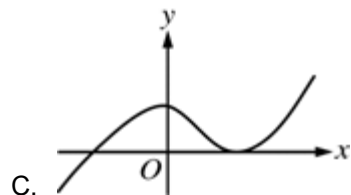
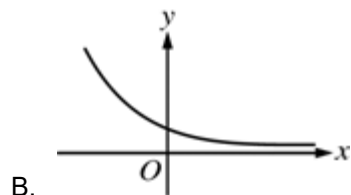
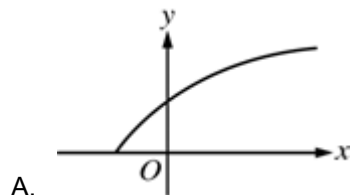
$$\begin{aligned}0 &= -2x + 5 \\ 2x &= 5 \\ x &= \frac{5}{2}\end{aligned}$$

The definite integral that gives the required area is:

$$\int_0^{5/2} (-2x + 5) dx$$

Options A, B and D are incorrect because the bounds of integration are incorrect.

9. If f is a differentiable real-valued function such that $f'(x) > 0$ and $f''(x) < 0$ for all $x \geq 0$, which of the following could be the graph of f in the xy -plane?



Answer

Option A is correct; if a function's first derivative $f'(x) > 0$ as $x > 0$, then the function must increase on the interval $(0, \infty)$; only graph A behaves in this way. Further, if a function's second derivative $f''(x) < 0$ as $x > 0$, then the function must be concave down on the interval $(0, \infty)$; again, only graph A behaves in this way. So graph A is the graph of f . **Option B is incorrect** because the graph shown is decreasing, not increasing. **Option C is incorrect** because the graph shown is increasing for some $x > 0$ and decreasing for other values. **Option D is incorrect** because it is concave up.

Domain III—Geometry and Measurement

Competency 012—The teacher understands geometries, in particular Euclidian geometry, as axiomatic systems.

10. In Euclidean geometry, the sum of the interior angles of a triangle is 180° . Let ABC be a triangle in elliptical geometry. Which of the following statements about the measures of angles A , B and C is true?

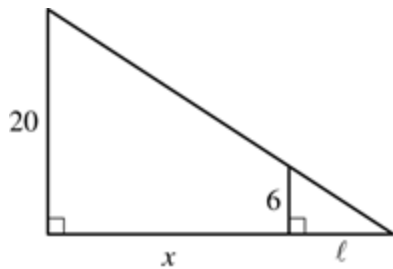
- A. $m\angle A + m\angle B + m\angle C = 180^\circ$
- B. $m\angle A + m\angle B + m\angle C < 180^\circ$
- C. $m\angle A + m\angle B + m\angle C > 180^\circ$
- D. $m\angle A + m\angle B + m\angle C = \infty$

Answer

Option C is correct because elliptical geometry is concerned with geometry of a sphere, and not a flat plane. In elliptical geometry, the "lines" are great circles on the sphere, i.e., circles whose centers are at the center of the sphere. The sides of a triangle drawn on a sphere are arcs of great circles. In elliptical geometry, the sum of the measures of a triangle is greater than 180° . **Option A is incorrect.** This statement is true in Euclidean geometry. **Option B is incorrect.** This statement is true in hyperbolic geometry. **Option D is incorrect.** This statement does not characterize any geometry.

Competency 013—The teacher understands the results, uses and applications of Euclidian geometry.

Use the figure below to answer the question that follows.



11. A student drew the figure shown to model a person 6 feet tall walking away from a lamppost 20 feet tall. Based on the figure, which of the following is true?

- A. $\frac{l}{6} = \frac{x+l}{20}$
- B. $\frac{l}{6} = \frac{x}{20}$
- C. $\frac{x+l}{6} = \frac{x}{20}$
- D. $\frac{x+l}{6} = \frac{l}{20}$

Answer

Option A is correct because the corresponding angles of the nested triangles are congruent and the triangles are similar. Because the right triangles are similar, the corresponding sides of the triangles must be proportional.

So the ratio of $\frac{\ell}{6}$ must equal the ratio of $\frac{x + \ell}{20}$. **Option B is incorrect** because x does not represent the length of the entire side of the triangle. **Option C is incorrect** because $x + \ell$ corresponds to 20, not 6. **Option D is incorrect** because l corresponds to 6, not 20 and $x + \ell$ corresponds to 20, not 6.

Competency 014—The teacher understands coordinate, transformational and vector geometry and their connections.

12. Let $v = (a_1, b_1, c_1)$ and $w = (a_2, b_2, c_2)$ be vectors in R^3 . If the dot product of two vectors is given by $v \cdot w = a_1a_2 + b_1b_2 + c_1c_2$, and $\|v\| = \sqrt{v \cdot v}$, then which of the following is equivalent to $(v + w) \cdot v - (v + w) \cdot w = 0$?

- A. $\|v\|^2 + v \cdot w + \|w\|^2 = 0$
- B. $\|v\|^2 + 2v \cdot w - \|w\|^2 = 0$
- C. $\|v\|^2 - 2v \cdot w + \|w\|^2 = 0$
- D. $\|v\|^2 - \|w\|^2 = 0$

Answer

Option D is correct because the first expression of the equation can be simplified to:

$$\begin{aligned} &(v + w) \cdot v \\ &= (a_1 + a_2, b_1 + b_2, c_1 + c_2) \cdot (a_1, b_1, c_1) \\ &= a_1(a_1 + a_2) + b_1(b_1 + b_2) + c_1(c_1 + c_2) \\ &= a_1a_1 + a_1a_2 + b_1b_1 + b_1b_2 + c_1c_1 + c_1c_2 \end{aligned}$$

Likewise, the second expression of the equation simplifies to:

$$a_2a_1 + a_2a_2 + b_2b_1 + b_2b_2 + c_2c_1 + c_2c_2$$

The left and right side of the equation share these values:

$$a_1a_2, b_1b_2, c_1c_2$$

So the equation shown can be simplified to:

$$a_1a_1 + b_1b_1 + c_1c_1 - (a_2a_2 + b_2b_2 + c_2c_2) = 0$$

or, $v \cdot v - w \cdot w = 0$, i.e., $v \cdot v = w \cdot w$.

Because $\|v\| = \sqrt{v \cdot v}$, then $\|v\|^2 = v \cdot v$, so:

$$\begin{aligned} v \cdot v &= w \cdot w \\ \|v\|^2 &= \|w\|^2 \\ \|v\|^2 - \|w\|^2 &= 0 \end{aligned}$$

Options A, B and C are incorrect because $v \cdot w = w \cdot v$. In the correct expansions, the terms with $v \cdot w$ can be combined so that the sums will have coefficients of 0.

Domain IV—Probability and Statistics

Competency 015—The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.

13. A calculus teacher gave an exam to 83 students where the highest possible score was 100. The teacher calculated that the average (arithmetic mean) score was 72.4, the median score was 72.0, the mode of the scores was 68.0 and the standard deviation of the scores was 12.0. There was a makeup exam for one student, and the student's score was 100. The teacher recalculated the average, median, mode, and standard deviation for the scores of all 84 students. Which of the following statements is true?

- A. The average and the standard deviation increased.
- B. The average, median and mode decreased.
- C. The median, mode and standard deviation decreased.
- D. The average, median and mode were unchanged.

Answer

Option A is correct because the average of the scores for the 83 students was:

$$\frac{\text{sum of 83 scores}}{83} = 72.4.$$

To find the new average:

$$\frac{100 + \text{sum of 83 scores}}{84} =$$
$$\frac{100 + 83(72.4)}{84} = 72.7 > 72.4$$

The standard deviation of a finite set of numbers is found by using the equation:

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2}$$

where N is the number of elements in the set, x is the value of the element, i is the order of the element, and μ is the average score of the set.

Substituting with the original set, it can be shown that:

$$12 = \sqrt{\frac{1}{83} \sum_{i=1}^{83} (x_i - 72.4)^2}$$
$$144 = \frac{1}{83} \sum_{i=1}^{83} (x_i - 72.4)^2$$
$$11952 = \sum_{i=1}^{83} (x_i - 72.4)^2$$

Substituting with the new set, it can also be shown that:

$$\sum_{i=1}^{84} (x_i - 72.7)^2 = (100 - 72.7)^2 + \sum_{i=1}^{83} (x_i - 72.7)^2$$
$$= 745.29 + \sum_{i=1}^{83} (x_i - 72.7)^2$$

Further:

$$\begin{aligned} & \sum_{i=1}^{83} (x_i - 72.7)^2 \\ &= \sum_{i=1}^{83} (x_i - 72.4 - 0.3)^2 \\ &= \sum_{i=1}^{83} [(x_i - 72.4)^2 - 2(0.3)(x_i - 72.4) + 0.09] \\ &= \sum_{i=1}^{83} [(x_i - 72.4)^2 - 0.6x_i + 43.53] \\ &= \sum_{i=1}^{83} (x_i - 72.4)^2 - 0.6 \sum_{i=1}^{83} x_i + \sum_{i=1}^{83} 43.53 \\ &= 11952 - 0.6(83)(72.4) + 43.53(83) \\ &= 11959.47 \end{aligned}$$

So the standard deviation of the 84-element set is:

$$\begin{aligned} \sigma &= \sqrt{\frac{1}{84} \sum_{i=1}^{84} (x_i - 72.7)^2} \\ &= \sqrt{\frac{1}{84} (745.29 + 11959.47)} \\ &\approx 12.298 \end{aligned}$$

The standard deviation increased with the inclusion of a score of 100 to the set. **Options B, C and D are incorrect** because the average score and the standard deviation each increased with the inclusion of the score of 100 to the set.

Competency 016—The teacher understands concepts and applications of probability.

14. A cube has its six faces colored red, white, blue, green, yellow and violet. Each of the six faces is equally likely to be face up when the cube is tossed. The cube is tossed twice. What is the probability that the red face will appear face up on the first toss, but will not appear face up on the second toss?

- A. $\frac{1}{36}$
- B. $\frac{5}{36}$
- C. $\frac{5}{12}$
- D. 1

Answer

Option B is correct. Because each of the six faces is equally likely to be face up when the cube is tossed, the probability that the red face will appear on the first toss is $\frac{1}{6}$. The probability that the red face will not appear is 1

$-\frac{1}{6} = \frac{5}{6}$. Because the two tosses are independent events, the probability that the red face will appear on the first toss, but not on the second, is $\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = \frac{5}{36}$. **Option A is incorrect** because the probability that the red face will not appear is $1 - \frac{1}{6} = \frac{5}{6}$, not $\frac{1}{6}$. **Option C is incorrect** because $\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = \frac{5}{36}$, not $\frac{5}{12}$. **Option D is incorrect**. The two tosses are independent events, and the probabilities are multiplied, not added.

Domain V—Mathematical Processes and Perspectives

Competency 018—The teacher understands mathematical reasoning and problem solving.

15. In 2012, the music industry in Country Y generated revenues of 900 million dollars in sound recordings. Jazz and blues sound recordings accounted for approximately 4.7 percent of the country's sale of sound recordings. The revenue from the sale of jazz and blues recordings in Country Y increased by 12.0 percent from 2010 to 2012. What was the approximate revenue from the sale of jazz and blues recordings in Country Y in 2010?

- A. 38 million dollars
- B. 60 million dollars
- C. 450 million dollars
- D. 803 million dollars

Answer

Option A is correct because in 2012 the revenue generated by jazz and blues sound-recording sales was 0.047×900 million dollars = 42.3 million dollars. Further, there was a 12.0% increase of sales from 2010 to 2012 in jazz and blues. Therefore, letting x represent the amount of sales in 2010, $1.12x = 42.3$. Solving for x shows that $x = 37.767\dots$, which is approximately 38 million dollars. So the approximate revenue of the sale of jazz and blues recordings in Country Y in 2010 was about 38 million dollars. **Options B, C and D are incorrect** because each is greater than 42.3 million dollars:

$$1.12(60 \text{ million}) = 67.2 \text{ million,}$$

$$1.12(450 \text{ million}) = 504 \text{ million, and}$$

$$1.12(803 \text{ million}) = 899.4 \text{ million.}$$

Use the equation below to answer the question that follows.

$$\cos(A + B) = \cos A + \cos B$$

16. A teacher gives his class the trigonometric equation above and asks the students to verify whether the equation is valid for all angles A and B . If the equation is not true for all angles A and B , he asks his students to give a counterexample that shows the equation to be false. If the equation is true, for all angles A and B , the students are asked to give a proof. By introducing the concept in this manner, the teacher

- A. understands the problem solving process.
- B. evaluates how well a mathematical model represents a real-world situation.

- C. recognizes and uses multiple representations of a mathematical concept.
- D. applies correct mathematical reasoning to make conjectures, and uses deductive methods to evaluate the validity of conjectures.

Answer

Option D is correct. By posing the question to the class and asking them to verify and, if the equation is not true, disprove the statement through counterexample, or prove the statement, the teacher is fostering in his students the opportunity to use deductive reasoning and the problem-solving process. This activity enables students to test conjectures using deductive reasoning and demonstrates the use of counterexamples. **Option A is incorrect** because the students are not asked to solve the problem but to determine the validity of the conjecture. **Option B is incorrect** because the equation is not derived from a real-life scenario. **Option C is incorrect** because the equation shown does not use multiple representations.

Domain VII—Scientific Inquiry and Processes

Competency 022—The teacher understands how to select and manage learning activities to ensure the safety of all students and the correct use and care of organisms, natural resources, materials, equipment and technologies.

17. During a laboratory investigation on density, the side of an aluminum cube is measured to be 2.49 cm. The volume of the cube should be recorded with the correct number of significant figures as which of the following?

- A. 15 cm^3
- B. 15.4 cm^3
- C. 15.44 cm^3
- D. 15.438 cm^3

Answer

Option B is correct because there are three significant figures in the measurement, and because the volume of the cube is calculated by multiplying the measurement, the correct number of the significant figures recorded for the volume should also be three. Therefore, the answer is 15.4 cm^3 . **Options A, C and D are incorrect** because they are not expressed with the correct number of significant figures.

Competency 023—The teacher understands the nature of science, the process of scientific inquiry and the unifying concepts that are common to all sciences.

18. Which of the following is a scientific inference?

- A. A mass scale is accurate to 0.01 grams.
- B. Two charged spheres are observed to attract each other.
- C. The periodic wobbling of a distant star suggests that a planet is in orbit around it.
- D. The average acceleration of a cart can be determined from the change in its velocity over a given time interval.

Answer

Option C is correct because it is a reasonable conclusion drawn from data or observation, which defines a scientific inference. **Option A is incorrect** because it is a specification for a measuring instrument. **Option B is incorrect** because it is an observation with no conclusion. **Option D is incorrect** because it describes a calculation that makes use of the definition of average acceleration.

Domain VIII—Physics

Competency 025—The teacher understands the description of motion in one and two dimensions.

19. Vector quantities include which of the following? Select all that apply.

- A. Force
- B. Kinetic energy
- C. Acceleration
- D. Electric field

Answer

Options A, C and D are correct because vector quantities have both a magnitude and a direction and this is true of force, acceleration and electric field. **Option B is incorrect** because kinetic energy has a magnitude but not a direction, which means that it is a scalar quantity.

Competency 027—The teacher understands the concepts of gravitational and electromagnetic forces in nature.

20. On the basis of Coulomb's law, which of the following is true about the electrostatic force that two charged objects X and Y exert on one another?

- A. The force is directly proportional to the sum of the squares of the charges of the two objects.
- B. The force is inversely proportional to the square of the distance between the two objects.
- C. If the charges on objects X and Y are of the same sign, then the force on object X is in the same direction as the force on object Y .
- D. If the charges on objects X and Y have opposite signs, then the force on object X is in the same direction as the force on object Y .

Answer

Option B is correct because, according to Coulomb's law, the electrostatic force between two charges is inversely proportional to the square of the distance between the charges. **Option A is incorrect** because, according to Coulomb's law, the electrostatic force between two charges is directly proportional to the product of the two charges, not the sum of the squares of the charges. **Options C and D are incorrect** because, according

to Newton's third law, the electrostatic forces that the charges exert on each other are always in opposite directions, regardless of the signs of the charges.

Competency 028—The teacher understands applications of electricity and magnetism.

21. Two $6\ \Omega$ resistors are connected in parallel to an ideal 24 V source. What is the current through the voltage source?

- A. 2 A
- B. 4 A
- C. 6 A
- D. 8 A

Answer

Option D is correct because the two resistors connected in parallel combine to have a $3\ \Omega$ equivalent resistance, while the ideal voltage source has zero internal resistance. Thus, by Ohm's law, the current through the 24 V source is equal to $(24/3)$ A, or 8 A. **Option A is incorrect** because it gives the result for two $6\ \Omega$ resistors connected in series, not parallel. **Option B is incorrect** because it gives the current through a single $6\ \Omega$ resistor. **Option C is incorrect** because it is just a statement of the resistor value expressed in amps instead of ohms.

Competency 029—The teacher understands the conservation of energy and momentum.

22. In an isolated system, which of the following quantities are conserved? Select all that apply.

- A. Velocity
- B. Linear momentum
- C. Angular momentum
- D. Energy

Answer

Options B, C and D are correct because, in an isolated system, linear momentum, angular momentum and energy are conserved quantities. **Option A is incorrect** because velocity is not a conserved quantity.

Competency 031—The teacher understands the characteristics and behavior of waves.

23. Which of the following is a consequence of refraction of light?

- A. The formation of an image after light passes through a lens
- B. The bending and spreading of light as it passes through a narrow slit
- C. The pattern formed when light passes through two narrow, parallel slits

D. The focusing of light into a small area after reflecting from a curved mirror

Answer

Option A is correct because the bending or refraction of light by the lens material results in the image formed by the lens. **Option B is incorrect** because it is diffraction, not refraction, that causes light to bend or spread when passing through a slit. **Option C is incorrect** because the pattern formed when light passes through two slits is due to interference, not refraction. **Option D is incorrect** because it refers to the focusing of light from a curved mirror as a result of reflection, not refraction.

Competency 032—The teacher understands the fundamental concepts of quantum physics.

24. According to the Bohr model of the hydrogen atom, what happens when an electron makes a transition from a lower energy level to a higher level?

- A. The nucleus loses kinetic energy.
- B. A photon is absorbed by the atom.
- C. A neutron decays into a proton, an electron, and a neutrino.
- D. A positron makes a transition from the higher level to the lower level.

Answer

Option B is correct because the Bohr model describes the hydrogen atom in terms of discrete stationary energy levels, and it is the transition of an electron from a lower energy level to a higher energy level that results from the absorption of a photon by the atom. **Option A is incorrect** because the kinetic energy of the nucleus is outside the scope of the Bohr model. **Option C is incorrect** because it describes neutron decay, which is outside the scope of the Bohr model. **Option D is incorrect** because hydrogen atoms do not contain positrons.

Domain IX—Chemistry

Competency 033—The teacher understands the characteristics of matter and atomic structure.

25. Of the following species, which has the largest ionic radius?

- A. Na^+
- B. Mg^{2+}
- C. Al^{3+}
- D. Cl^-

Answer

Option D is correct because the additional electron in Cl^- increases the electron-electron repulsion and therefore the ionic radius is larger than the atomic radius. Cl^- has the same electron configuration as Ar. **Options A, B and C are incorrect** because Na^+ , Mg^{2+} , and Al^{3+} have the same electron configuration as Ne. The

attractive forces exerted by the nucleus on the remaining electrons cause contraction, so the ionic radii decrease in the following order: $\text{Na}^+ > \text{Mg}^{2+} > \text{Al}^{3+}$.

Competency 035—The teacher understands the properties and characteristics of ionic and covalent bonds.

26. Which of the following best describes the molecular geometry of formaldehyde H_2CO ?

- A. Tetrahedral
- B. Trigonal planar
- C. Trigonal pyramidal
- D. T-shaped

Answer

Option B is correct because formaldehyde has 3 bonding electron domains. According to the VSEPR model, the molecular geometry is trigonal planar. **Option A is incorrect** because molecules with 4 bonding electron domains have tetrahedral geometry. **Option C is incorrect** because molecules with 3 bonding electron domains and one nonbonding electron domain have trigonal pyramidal geometry. **Option D is incorrect** because molecules with 3 bonding electron domains and 2 nonbonding electron domains have T-shaped geometry.

Competency 036—The teacher understands and interprets chemical equations and chemical reactions.

27. Which of the following is the balanced equation for the displacement reaction of aluminum with silver nitrate?

- A. $\text{Al} + \text{AgNO}_3 \rightarrow \text{AlNO}_3 + \text{Ag}$
- B. $\text{Al} + 3 \text{AgNO}_3 \rightarrow \text{Al}(\text{NO}_3)_3 + 3 \text{Ag}$
- C. $\text{Al} + \text{AgNO}_3 \rightarrow \text{AlN} + \text{AgO}_3$
- D. $\text{Al} + \text{Ag}_3\text{N} \rightarrow \text{AlN} + 3 \text{Ag}$

Answer

Option B is correct because the reaction of aluminum with silver nitrate forms $\text{Al}(\text{NO}_3)_3$ and Ag. The formula for silver nitrate is AgNO_3 . The equation is balanced. **Option A is incorrect** because the formula for aluminum nitrate is not correctly represented. **Option C is incorrect** because the incorrect products are formed. **Option D is incorrect** because silver nitrate and aluminum nitrate are incorrectly represented.

Competency 037—The teacher understands types and properties of solutions.

28. Of the following solutions, which has the highest electrical conductivity?

- A. 1 M NaCl
- B. 1 M HCH_3CO_2

- C. 1 M HCl
- D. 1 M Na₂SO₄

Answer

Option D is correct because 1 M Na₂SO₄ dissociates to produce 3 moles per liter of ions. **Option A is incorrect** because 1 M NaCl dissociates to produce 2 moles per liter of ions. **Option B is incorrect** because HCH₃CO₂, a weak acid, only partly dissociates in aqueous solutions and is a weak electrolyte. **Option C is incorrect** because 1 M HCl, a strong acid, dissociates to produce 2 moles per liter of ions.

Competency 038—The teacher understands energy transformations that occur in physical and chemical processes.

Use the table below to answer the question that follows.

Specific heat of ice	2.03 J/g·°C
Specific heat of water	4.18 J/g·°C
Heat of fusion of ice	334 J/g

29. Based on the data in the table above, what is the change in heat content of 100. g of water when it is cooled from 20.0°C to –20°C?

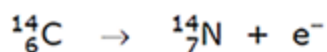
- A. –45,820 J
- B. –12,420 J
- C. +33,400 J
- D. +45,820 J

Answer

Option A is correct because the change in heat content of 100. g of water when it is cooled from 20°C to ice at –20°C is the sum of the heat released by the cooling of the water, the freezing of the water, and the cooling of the ice. **Option B is incorrect** because it is the sum of the heat released by cooling of the water and ice but does not include the heat released during the freezing process. **Option C is incorrect** because it is the amount of energy required to melt ice at 0°C. **Option D is incorrect** because it is the change in heat content for the reverse process of warming ice at –20°C to water at 20°C.

Competency 039—The teacher understands nuclear fission, nuclear fusion and nuclear reactions.

Use the equation below to answer the question that follows.



30. The isotope carbon-14 decays according to the equation shown above. The transformation is an example of

- A. alpha decay.

- B. beta decay.
- C. gamma decay.
- D. x-ray emission.

Answer

Option B is correct because beta-minus decay is the process in which a neutron is converted to a proton and an electron, a beta particle, is emitted. **Option A is incorrect** because alpha decay is the process in which a helium ion, an alpha particle, is emitted from the nucleus and a lighter nuclide is produced. **Option C is incorrect** because gamma decay is the process in which electromagnetic radiation with high frequency and high energy is emitted from atomic nuclei and is indicated by the Greek symbol gamma γ . **Option D is incorrect** because x-rays are electromagnetic radiation emitted by electrons.

Competency 040—The teacher understands oxidation and reduction reactions.

Use the table below to answer the question that follows.

Reduction Half-Reaction	E° (V)
$\text{Ag}^+(\text{aq}) + \text{e}^- \rightarrow \text{Ag}(\text{s})$	+0.80
$\text{Cu}^{2+}(\text{aq}) + 2\text{e}^- \rightarrow \text{Cu}(\text{s})$	+0.34
$\text{Fe}^{2+}(\text{aq}) + 2\text{e}^- \rightarrow \text{Fe}(\text{s})$	-0.44
$\text{Zn}^{2+}(\text{aq}) + 2\text{e}^- \rightarrow \text{Zn}(\text{s})$	-0.76

31. Based on the standard reduction potentials in the table above, which of the following reactions will occur spontaneously?

- A. $2 \text{Ag}(\text{s}) + \text{Fe}^{2+}(\text{aq}) \rightarrow 2 \text{Ag}^+(\text{aq}) + \text{Fe}(\text{s})$
- B. $\text{Cu}(\text{s}) + \text{Fe}^{2+}(\text{aq}) \rightarrow \text{Cu}^{2+}(\text{aq}) + \text{Fe}(\text{s})$
- C. $\text{Fe}(\text{s}) + \text{Zn}^{2+}(\text{aq}) \rightarrow \text{Fe}^{2+}(\text{aq}) + \text{Zn}(\text{s})$
- D. $\text{Zn}(\text{s}) + \text{Cu}^{2+}(\text{aq}) \rightarrow \text{Zn}^{2+}(\text{aq}) + \text{Cu}(\text{s})$

Answer

Option D is correct because the net potential for the reaction is $E^\circ = [+0.34 \text{ V} + (+0.76 \text{ V})] = +1.10 \text{ V}$. The sign of the standard reduction potential for the species that is oxidized, Zn, is reversed before summing the standard reduction potentials for the two half-reactions. A net potential that is positive indicates that the reaction is spontaneous. **Option A is incorrect** because the net potential for the reaction is -1.24 V . **Option B is incorrect** because the net potential for the reaction is -0.78 V . **Option C is incorrect** because the net potential for the reaction is -0.32 V .

Domain X—Scientific Learning, Instruction and Assessment

Competency 042—The teacher understands research-based theoretical and practical knowledge about teaching science, how students learn science and the role of scientific inquiry in science instruction.

32. Which of the following is an example of inquiry-based science learning?

- A. A group of students uses small carts to study whether kinetic energy is conserved when they collide.
- B. A classroom of students watches a video on the similarities and distinctions between work and impulse.
- C. A teacher gives a presentation on the role heating and work play in the conservation of energy.
- D. A student conducts a literature search on the role of entropy in thermodynamic systems.

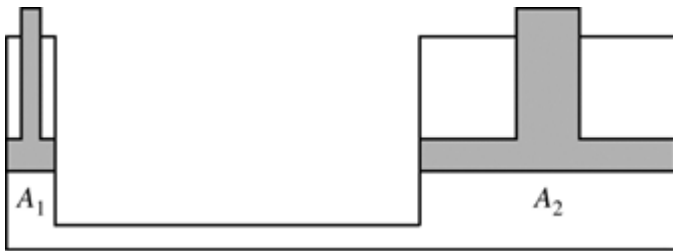
Answer

Option A is correct because inquiry-based science involves students doing a hands-on experiment to answer a question. **Option B is incorrect** because videos can be helpful, but they are not elements of inquiry-based learning. **Option C is incorrect** because teacher presentations are important, but they are not an element of inquiry-based learning. **Option D is incorrect** because searching and reading literature can have value, but they are not an element of inquiry-based learning.

Domain XI—The Engineering Method

Competency 044—The teacher has a working knowledge of engineering fundamentals.

Use the figure below to answer the question that follows.



33. A technician designs a hydraulic system to raise heavy objects. The cross-sectional area of the smaller piston is A_1 and the cross-sectional area A_2 of the larger piston is 10 times that of the smaller piston. The system is filled with an incompressible fluid. If the mechanical advantage of the system is 10, how much force must be applied to lift a 200-kilogram mass and on which piston should the force be applied? (Assume that the gravitational acceleration is 10 meters per second squared.)

- A. A force of 200 newtons must be applied to the larger piston.
- B. A force of 200 newtons must be applied to the smaller piston.
- C. A force of 20 newtons must be applied to the larger piston.
- D. A force of 20 newtons must be applied to the smaller piston.

Answer

Option B is correct because the hydraulic press is based on Pascal's law: in a fluid at rest in a closed container, a pressure change at one point is transmitted — without loss — to every other point in the fluid, and is also transmitted onto the walls of the container. That means pressure is the same on both pistons. Moreover, pressure is equal to the force divided by the area on which it acts.

Let m be the mass, g the gravitational acceleration, F_1 the force on piston 1, F_2 the force on piston 2, A_1 the area of piston 1, A_2 the area of piston 2, $p_1 = \frac{F_1}{A_1}$ the pressure on piston 1 and $p_2 = \frac{F_2}{A_2}$ the pressure on piston 2.

Because the pressure is equal and it is given that $A_2 = 10A_1$, we have:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$
$$\frac{F_1}{A_1} = \frac{F_2}{10A_1}$$
$$10F_1 = F_2$$

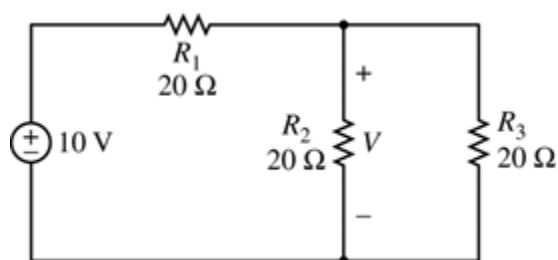
The technician will act on piston 1 in order to exert less force. To lift a 200 kilogram mass with force F_1 , first calculate F_2 , then solve for F_1 .

$$F_2 = (200 \text{ kg})\left(10 \frac{\text{m}}{\text{s}^2}\right) = 2000 \text{ N.}$$

$$F_1 = \frac{F_2}{10} = 200 \text{ N.}$$

Option A is incorrect because the force is right but the piston is wrong. **Option C is incorrect** because both the force and the piston are wrong. **Option D is incorrect** because the piston is right but the force is wrong.

Use the closed electrical circuit below to answer the question that follows.



34. The closed electrical circuit shown consists of a generator and three resistors, R_1 , R_2 and R_3 . The electromotive force of the generator is 10 volts. Which of the following is the second resistor's voltage, V ?

- A. 3.33 V
- B. 5.00 V
- C. 6.67 V
- D. 10.00 V

Answer

Option A is correct because of Ohm's law, which relates the voltage (V), the current (I) and the resistance (R) according to the formula $V = RI$.

First resolve the complexity of the circuit by finding the resistance for the parallel circuits using the formula:

$$\begin{aligned}\frac{1}{R_{2-3}} &= \frac{1}{R_2} + \frac{1}{R_3} \\ &= \frac{1}{20} + \frac{1}{20} \\ &= \frac{1}{10} \text{ ohms}\end{aligned}$$

Then find the resistance of the circuit by adding R_1 and R_{2-3} , $20 + 10 = 30$ ohms. Using Ohm's law and the fact that EMF of the generator equals the voltage of the circuit, $I = \frac{V}{R} = \frac{10}{30} = \frac{1}{3}$.

Further, as the current passes through this circuit it splits in half when it passes through the parallel circuits. This is because the resistance and voltage values are equal.

Applying Ohm's Law a second time:

$$V_2 = R_2 I_2 = 20 \left(\frac{1}{2} \right) \left(\frac{1}{3} \right) \approx 3.33 \text{ V.}$$

Options B, C and D are incorrect because the values of the voltages are wrong.

35. A broken down car, with mass 1859 kilograms, is accelerated southward at 1.32 meters per second squared when pulled by a tow truck. Which of the following is closest to the force of friction if the tow truck exerts 2.84 kilonewtons, southward on the car?

- A. 0.386 kilonewtons
- B. 0.842 kilonewtons
- C. 5.293 kilonewtons
- D. 2840 kilonewtons

Answer

Option A is correct because Newton's second law of motion states that the sum of the forces, F , on an object is equal to the total mass, m , of that object multiplied by the acceleration, a , that force gives to the object: $F = ma$.

The total force applied to the car is its mass, 1859 kilograms, times its acceleration, 1.32 meters per second squared:

$$\begin{aligned}F &= (1859)(1.32) = 2453.88 \text{ newtons} \\ &= 2.45 \text{ kilonewtons}\end{aligned}$$

In the system there are two forces: the tow truck force, F_t , and the friction, F_f .

$$F = F_t + F_f, \text{ then } F_f = F - F_t = 2.454 - 2.840 = -0.386 \text{ kN}$$

The minus sign indicates that the friction forces are in the opposite direction of the motion and the direction of the total force.

Option B is incorrect because the correct value of F_f is 0.386 kN.

Option C is incorrect because 5.293 kN is the result of the algebraic sum of F_f and F , but $F_f = F - F_t$.

Option D is incorrect because the correct value of friction F_f is 0.386 kN. Furthermore the friction must be lower than the tow truck force to have the car moving. With a friction of 2840 kN the car cannot move.

Competency 046—The teacher understands basic principles of information technology and computers and the role of information technology.

36. A wireless communication system has been set up between two locations and is said to be problematic because it is band-limited. Which of the following is the most likely cause of the problem?

- A. It takes too long to encode the bits that are to be transmitted
- B. There is not enough computer memory in the system
- C. There is not enough dedicated transmission bandwidth to send the information quickly enough
- D. There are types of transmissions that are forbidden due to security risks

Answer

Option C is correct because "band-limited" is a common term meaning that there is not enough transmission frequency space to quickly and efficiently send information. **Option A is incorrect** because the time necessary for the transmission of data does not have a threshold that once exceeded may be considered a problem by the system. **Option B is incorrect** because a "band-limited problem" is not correlated to a computer memory problem. **Option D is incorrect.** When there are types of transmissions that are forbidden due to security risks, it's the firewall system or the security program that stops the transmission.

Domain XII—The Engineering Profession

Competency 050—The teacher understands the concept of teaming, demonstrates knowledge of careers in engineering and understands the legal and ethical requirements of the engineering profession.

Use the table below to answer the question that follows.

Maximum repayment period	60 months
Maximum monthly payment	\$20,000
Maximum total repayment	\$1,000,000

37. The board of directors of a small engineering firm has approved a long-term investment in one of their projects that will require securing a \$500,000 loan. The guidelines for the loan are listed in the table shown. Which of the following options represents an offer from a commercial banking institution that meets the criteria set by the board?

- A. 48 months, at 12% annual interest, compounded quarterly

- B. 30 months, at 24% annual interest, compounded quarterly
- C. 36 months, at 3% simple and 36% annual interest
- D. 24 months, at 4% simple and 48% annual interest

Answer

Option A is correct because the use of the compound interest formula

$A = P \left(1 + \frac{r}{n}\right)^{nt}$, where P is the loan amount, \$500,000, t is the maximum repayment period of 4 years, r is the annual interest rate of 12% or 0.12, and n is 4 because the interest is compounded quarterly.

The total repayment is

$$\begin{aligned} A &= 500,000 \left(1 + \frac{0.12}{4}\right)^{4(4)} \\ &= 500,000(1.03)^{16} \\ &= 802,353.22 \end{aligned}$$

Comparing restrictions, option A's repayment period is under 60 months. Further, the monthly payment is $\frac{802,353.22}{48} = 16,715.69$, which is under the maximum monthly payment of \$20,000. Finally, the engineering firm will have to pay \$802,353.22 over the course of the 48 months, which is less than maximum total repayment of \$1,000,000.

Option B is incorrect because it does not meet the second restriction — maximum monthly payment — set by the board. Using the compound interest formula, the monthly payment would be $\frac{895,423.85}{30} = \$29,847.46$, which

is more than \$20,000 per month. **Option C is incorrect** because it does not meet either the second or third restriction. To find the total repayment, use both the simple interest and simple annual interest formulas:

$$I_{\text{simple}} = Pi$$

$$A_{\text{simple}} = P(1 + tr)$$

$$\text{Total} = A_{\text{simple}} + I_{\text{simple}}$$

$$\begin{aligned} \text{Total} &= A_{\text{simple}} + I_{\text{simple}} \\ &= 500,000(1 + (3)(0.36)) + 500,000(0.03) \\ &= 1,055,000 \end{aligned}$$

and the monthly payment is $\frac{1,055,000}{36} = 29,305.60$.

Option D is incorrect because it does not meet the second restriction set by the board. The total is given by:

$$\begin{aligned} \text{Total} &= A_{\text{simple}} + I_{\text{simple}} \\ &= 500,000(1 + (2)(0.48)) + 500,000(0.04) \\ &= 1,000,000 \end{aligned}$$

The monthly payment would be: $\frac{1,000,000}{24} = 41,666.67$.

38. An engineering manager who supervises a team of twelve people has been assigned to generate reports from all four sectors of production. The manager is newly appointed and has been asked by the boss to practice delegation. Which of the following actions are most appropriate and efficient to get the reports completed?

- A. Personally writing all the reports
- B. Meet with senior members of the team to determine which team members are best suited to write the reports, and then let the senior members assign the reports to whoever is appropriate. The new manager will review and edit the reports before sending to the boss.
- C. Assigning one report to each of the twelve team members and then personally reviewing, consolidating and rewriting all the reports to correct any important omissions
- D. Assigning all four reports to one randomly selected team member, assuming that all members are well qualified to write a report

Answer

Option B is correct because in order to practice delegation, the manager should share responsibilities. The most efficient way for the new manager to have the reports generated and to practice delegation is to meet with senior members of the team who know about the other employees' skills and then let senior members of the team assign the report writing to the most qualified individuals. It is a good practice for a manager to review and edit any report being sent to senior management. **Option A is incorrect** because the new manager does not delegate anything or share responsibilities in this process. **Option C is incorrect** because this solution is inefficient. Assigning one report to each of the twelve team members without regard to experience or ability may lead to omissions or may require additional time for completion. Further, by reviewing, consolidating, and editing each report, the manager may spend time that could be better allocated to other tasks. **Option D is incorrect** because the process is inefficient. Randomly assigning the report to one team member without regard to experience, ability or workload may lead to omissions, errors and an incomplete report.

39. An engineering firm has been hired by the city to construct a bridge over a newly formed irrigation channel. The main professional engineer (PE) is out on medical leave, and another engineer has taken his place as temporary PE. The temporary PE has just been recently certified in Civil Transportation by the National Council of Examiners for Engineering and Surveying (NCEES) and he is competent but unsure of himself in certain areas of the safety requirements for the bridge. He would prefer to delay signoff until the experienced PE comes back, but an official from the city asks the temporary PE if it is possible to speed up the sign-off process. By signing off without having the experienced PE review the plan, this NCEES-licensed PE will be violating his obligation to which of the following?

- A. To society
- B. To society, his employer and his clients
- C. To other licensees
- D. To society, his employer, his clients and other licensees

Answer

Option D is correct because by certifying work in an area where the engineer is unsure of himself, he is putting society at risk, putting the reputation of his employer at risk, putting his clients at risk and by violating the professional code of ethics is putting the profession at risk. Therefore he is violating his obligation to all four

entities mentioned in the answer choices. **Options A, B and C are incorrect** because they are not complete answers. Option D is the only complete answer that includes all groups affected by the engineer's choice.

40. A chemical engineering company, ChemA, has developed a new type of chemical process and has patented a new piece of equipment to run the process. An engineer owns a small business that provides expendable parts and supplies for use with the equipment. According to the code of ethics, when asked at a press conference to give his opinion on the effectiveness of the new equipment over equipment using older techniques, the engineer should

- A. refrain from making any comments since he has the potential to profit from the acceptance of the new equipment by the users.
- B. make statements that are only highly technical in nature in order to confuse the press.
- C. disclose his relationship with the company that manufactures the processing equipment before commenting or giving an opinion.
- D. read a statement that has been prepared by ChemA specifically for the purposes of responding to questions from the press.

Answer

Option C is correct according to the code of ethics of the National Society of Professional Engineers. **Option A is incorrect** because according to the code of ethics of the National Society of Professional Engineers, honesty and information sharing must supersede profit. **Option B is incorrect** because according to the code of ethics of the National Society of Professional Engineers, everyone must encourage honesty and information sharing. It is ethically wrong for an engineer to intentionally confuse the press or the public. **Option D is incorrect** because reading a statement that has been prepared by ChemA instead of giving a personal opinion could indicate that he is withholding his real opinion in favor of ChemA and personal profit.

Competency 051—The teacher knows how to provide a safe and productive learning environment.

41. A good lab instructor promotes a policy of safety awareness and helps students, faculty and staff develop positive attitudes toward safety. Which of the following is the best procedure to follow to help students prevent accidents in the lab?

- A. Fully explaining all safety considerations to students at the beginning of each lab
- B. Demonstrating safety procedures and identifying potential hazards before students begin their work and identifying hazards and applying safety procedures as appropriate throughout the lab
- C. Posting and distributing written directions or warnings, when appropriate, about the safe handling of materials and equipment used in instruction
- D. Regularly administering safety quizzes to students to ensure that safe procedures are followed

Answer

Option B is correct because it is the best way to prevent accidents. Practical activity stimulates interest and attention. Safety procedures are more easily learned if developed practically rather than theoretically. Identifying potential hazards and demonstrating the corresponding safety procedures will allow everyone to act properly under dangerous conditions. As safety hazards arise, they must be addressed immediately. **Option A is in**

correct because while fully explaining applicable safety considerations is a good practice, the process described in option B is more effective if the instructor wants to raise safety awareness for each practical activity. **Options C and D are incorrect** because written materials and quizzes can help reinforce safety procedures, but they may not address practical daily activities.

42. A group of students are participating in a lab activity in which they design and test different types of electrical circuits. Which of the following safety procedures is most important for the students to follow?

- A. Removing drinks from the laboratory area prior to the activity
- B. Wearing safety goggles and aprons while in the lab area
- C. Disposing of chewing gum before starting the lab activity
- D. Keeping loose hair and clothing tied back during the lab activity

Answer

Option A is correct because water and other liquids can conduct electricity. Removing drinks from the lab area ensures that an accidental spill will not lead to the student's receiving an electrical shock. **Option B is incorrect** because although wearing safety goggles and aprons is a safety procedure, it is less specific and important during an electrical experiment compared with the danger of the liquids' electrical conductivity. **Option C is incorrect.** Although disposing of chewing gum is a safety procedure, it is less specific and important during an electrical experiment compared with the danger of the liquids' electrical conductivity. **Option D is incorrect** because although keeping loose hair and clothing tied back during the lab activity is a safety procedure, it is less specific and important during an electrical experiment compared with the danger of the liquids' electrical conductivity. Clothing and hair do not transmit electricity very well so they are not electrical risks. They may however reduce the field of view and have negative interactions with the material and equipment on the worktable.

Use the tables below to answer the question that follows.

Device	Power Rating
Vacuum	1360 W
Spotlight	720 W

Device Amperage Rating	Power Rating
1–7 amps	16 gauge
8–10 amps	14 gauge
11–14 amps	12 gauge
14+ amps	10 gauge

43. A technician is setting up the machine shop lab and needs to use a spotlight and shop vacuum, with the requirements shown. If the lab has a 120V source, and the technician is plugging both appliances into the same extension cord in parallel, what is the minimum gauge wire the setup requires?

- A. 16 gauge

- B. 14 gauge
- C. 12 gauge
- D. 10 gauge

Answer

Option D is correct because the appliances are in parallel, the total power required is the sum of both devices, which totals 2080W. The amperage (A) is calculated by dividing the power (P) by the voltage (V): $A = \frac{P}{V} = \frac{2080}{120} = 17.33$ amps. Therefore a minimum of 10 gauge wire is necessary. **Option A is incorrect** because 16 gauge wire is the minimum wire for 1–7 amps. The total required amperage is 17.33 amps, which requires a minimum of 10 gauge — not 16 gauge — wire. **Option B is incorrect** because 14 gauge wire is the minimum wire for 8–10 amps. The total required amperage is 17.33 amps, which requires a minimum of 10 gauge — not 14 gauge — wire. **Option C is incorrect** because 12 gauge wire is the minimum wire for 11–14 amps. The total required amperage is 17.33 amps, which requires a minimum of 10 gauge — not 12 gauge — wire.

Competency 052—The teacher understands the importance of professional development and how to apply engineering knowledge to plan, implement and assess student learning.

44. A group of eighth grade students received the following instruction from their teacher:

"Using less than 15 feet of masking tape and 110 paper straws, design a bridge that will span the two tables in the classroom and hold the weight of the electric pencil sharpener. Test to determine if plastic straws are better for building the structure than paper straws. Then try wooden frozen-dessert sticks instead of straws."

Which of the following best describes this type of activity?

- A. An open-ended, project-based activity
- B. A focused, project-based activity
- C. A controlled experiment
- D. A qualitative inquiry

Answer

Option A is correct because the activity proposed by the teacher is an intentional and planned learning experience that helps students to develop new skills and learn new information, while maintaining the positive association with play. Therefore it is a project-based activity. Because there are many ways to complete the activity and many potential outcomes, it is also open-ended. **Option B is incorrect** because even if the activity proposed by the teacher is a project-based activity, the teacher does not explain how to build the bridge. Since the students are free to devise their personal solution, the activity is not focused. **Option C is incorrect** because the activity proposed by the teacher is not based on a defined repeatable procedure and the consequent logical analysis of the results. **Option D is incorrect** because qualitative inquiry means studying cases in their natural settings and then subjecting the resulting data to analytical induction.